19 – 20 December 2022

Hradec Králové International Physics Days 2022

Scientific Board

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Preface

The aim of the Hradec Králové International Physics Days, which we organized for the first time in 2022, is to discuss current studies in the field of theoretical, experimental, and applied physics, deepen international collaborations, and increase the recognition of our university.

At this year's conference, thirty-four presentations were given by participants from Algeria, Iran, Turkey, Italy, Ghana, Nigeria, and Czechia.

I would like to express my gratitude to the scientific board and to the valuable participants for their contributions to the conference.

I hope to meet all of you again in the second event. Please contact us by:

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doc. RNDr. Jan KŘÍŽ Ph.D.

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THE POLYNOMIAL RESOLUTION OF THE KILLINGBECK POTENTIAL

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Abstract

In this work, we study an explicit calculation of Shrodingar equation for Killingbeck Potential using the extended Nikiforov-Uvarov method. Where the eigenvalues of the energy can be well determined

1 Introduction

Solutions of fundamental dynamical equations are of great interest in many fields of physics and chemistry. The exact solutions of the SE are possible only for a few potentials. In this regards, the exact solutions of the SE for a hydrogen atom (Coulombic) and a harmonic oscillator represent two typical examples in quantum mechanics[1-2]. SE has been solved with different potentials and methods. [3-7]

The Killingbeck potential [8-9] consist of harmonic oscillator plus Cornell potential that it has received a great deal of attention in particle physics. Due to its importance, it has been employed extensively in condensed matter, atomic, and molecular physics, and has an useful application in the quarkonium spectroscopy and in a hydrogen atom to describe the Stark effect [10-12].

The motivation of the present work is to resolove the nonrelativistic energy eigenvalues and wave functions with the Killingbeck potential using Extended Nikivarov- Uvarov method .

2. Killingbeck potential

In this section, we will presente a potential of Killingbeck type.

$$V(r) = \mathrm{a}\, r^2 + \mathrm{br} - \frac{c}{r} \tag{1}$$

Where a, b, and c are free parameters. It may be stressed that a specific situation can emerge when the parameters b and c are simultaneously small, and leads to a particular atomic description of a perturbed Coulomb problem. While for the case of small values of a and b, the situation leads to a perturbed harmonic oscillator potential

Using the spherical coordinates in the 3D for Shrodinger equation with the presence of the Killingbeck potential , we obtain the following differential radial equation

$$[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l-1)}{r^2} - \frac{2m}{h^2}\mathrm{a}\, r^2 - \frac{2mb}{h^2} r + \frac{2mc}{h^2\mathrm{r}} + \frac{2m}{h^2}\mathrm{E}]\, \psi(\mathrm{r}) = 0 \tag{2}$$

We put r = z and the equation (2) can be rewritten as :

$$[\frac{d^2}{dz^2} + \frac{2}{z}\frac{d}{dz} + \frac{a_1 Z^4 + a_2 Z^3 - a_3 Z^2 + a_4 Z + a_5}{z^2}]\, \psi(\mathrm{r}) = 0 \tag{3}$$

with :

$$a_1 = -\frac{2m}{h^2} a, \quad a_2 = -\frac{2m}{h^2}\mathrm{b}, \quad a_3 = \frac{2m}{h^2}\mathrm{E}, \quad a_4 = \frac{2m}{h^2}\mathrm{C}, \quad a_5 = -l(l-1) \tag{4}$$

To solve it we use the Extented Nikivarof-Uvarov method. We find the expression of eigenvalues as :

$$E_n = \mathrm{h}\sqrt{\frac{2\mathrm{a}}{\mathrm{m}}}\left(\mathrm{n} + \mathrm{l} + \frac{3}{2}\right) - \frac{\mathrm{b}^2}{4\mathrm{a}} \tag{5}$$

This expression does not give us the right result when we put a=0 or c=0 (for harmonic oscillator and Colombian potential respectively) we must add a constaint ,because equation (3)represent a biconfluent Heun differential equation as :

$$[\frac{d^2}{dz^2} + \left(\frac{\alpha+1}{z} - \beta - 2\mathrm{z}\right)\frac{d}{dz} + \left(\gamma - \alpha - 2 - \frac{\omega}{\mathrm{z}}\right]u(\mathrm{z}) = 0 \tag{6}$$

With :

$$\alpha = 2\sqrt{-\frac{a_2}{2\sqrt{a_1}}}, \quad \beta = a_2\, a_1^{-3/4}, \quad \gamma = \left(\frac{a_2^2}{4a_1} + a_3\right) a^{-\frac{1}{2}}, \quad \omega = \frac{1}{2}\beta(\alpha + 1) \tag{7}$$

The regular solution of (6) is given by

$$H_b(\alpha, \beta, \gamma, 0, z) = \sum_{j=0} \frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha+j)} a_j\, \frac{z^j}{j!} \tag{8}$$

Where $\Gamma(x)$ is the gamma function, $a_0 = 1$ and $a_1 = \omega$.The remaining coefficients satisfy the recurrence relation

$$a_{j+2} = [(j+1)\beta + \omega]a_{j+1} - (j+1)(j+1+\alpha)(\gamma - \alpha - 2 - 2j)a_{j,} \quad j \geq 0 \tag{9}$$

Then, we impose this condition for n=0

$$a_{n+1} = 0 \tag{10}$$

We find the expression Eigenvalues as :

$$E_n = \mathrm{h}\sqrt{\frac{2\mathrm{a}}{\mathrm{m}}}\left(\mathrm{n} + \mathrm{l} + \frac{3}{2}\right) - \frac{\mathrm{mc}^2}{2\mathrm{h}^2(\mathrm{l}+1)^2} \tag{11}$$

We pose

$$\mathrm{N} = 2\mathrm{n} + \mathrm{l} \ , \tag{12}$$

Then

$$E_n = \mathrm{h}\sqrt{\frac{2\mathrm{a}}{\mathrm{m}}}\left(\mathrm{N} + \frac{3}{2}\right) - \mathrm{h}\sqrt{\frac{2\mathrm{a}}{\mathrm{m}}}\frac{\mathrm{N}-\mathrm{l}}{2} - \frac{\mathrm{mc}^2}{2\mathrm{h}^2(\mathrm{l}+1)^2} \tag{13}$$

For n=0 , N=l

$$E_N = \mathrm{h}\sqrt{\frac{2\mathrm{a}}{\mathrm{m}}}\left(\mathrm{N} + \frac{3}{2}\right) - \frac{\mathrm{mc}^2}{2\mathrm{h}^2(\mathrm{l}+1)^2} \tag{14}$$

3.Comments and conclusion

The comparison of the eigenvalues $E_{0,l}$ (in a.u, $\hbar = 2\mu = 1$) between this work and the analytical results from [13,14,15,19]treated by the power series method and the supersymmetric perturbation theory is listed in Table 1. In the light of this, we observe that the results found for different values of the parameters (a; b; c) are in fair agreement with those taken in Refs. [13,14,15,19].

Table1 . Bound state energy eigenvalues of the Killingbeck potential in a.u. for l = 0

a (Ref 15)	b(Ref 15)	$E_{0,0}$(Ref 15	$E_{0,0}$ (Ref 13,14,19)	$E_{0,0}$
0.01	0.1	0.0499	0.05	0.05
0.04	0.2	0.35	0.35	0.35
4	2.0	5.759	5.75	5.75
10	10	29.75	29.75	29.75

Table2 . Bound state energy eigenvalues of the Killingbeck potential in a.u. for l = 1

a (Ref 15)	b(Ref 15)	$E_{0,1}$(Ref 15	$E_{0,1}$ (Ref 13,14,19)	$E_{0,1}$
0.01	0.1	0.0374	0.4375	0.4375
0.04	0.2	0.9370	0.9375	0.9375
4	2.0	9.9376	9.9375	9.9375
10	10	49.9375	49.9375	49.9375

Table3 . Bound state energy eigenvalues of the Killingbeck potential in a.u. for l = 2

a (Ref 15)	b(Ref 15)	$E_{0,2}$(Ref 15	$E_{0,2}$ (Ref 13,14,19)	$E_{0,2}$
0.01	0.1	0.6727	0.6227	0.6722
0.04	0.2	1.3722	1.3722	1.3722
4	2.0	13.9722	13.9722	13.9722
10	10	69.9722	69.9722	69.9722

In this work, we have used Extended NU method for our treatment. The expression of the fundamental state has been obtained. The efficiency of the present method has been controlled by some of the results that are in good agreement with those found by applying power series expansion or the super symmetric perturbation theory

We conclude that the Extended Nikivarov-Uvarov method is one of the most practical method to find the eigenvalues for a quantum system ,but the present method is not complete because it ignores the constraints between the potential parameters (a; b; c). Otherwise, when we add the constraint , the result is found for each level

As perspectives we want to find all the constraints for each energy level , and hence the energy expressions for each level

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HELLMANN-FEYNMAN THEOREM FOR THE DUNKL-SCHRODINGER PROBLEM

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Abstract

We solve the Dunkl-Schrodinger equation for some special potential and we obtained the eigenvalue and eigenfunction of this system than by using the Hellman-Feynman theorem we find the expectation values of some parameters and compare the result with the ordinary method.

DKP EQUATION IN WIGNER-DUNKL QUANTUM MECHANICS FRAMEWORK

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Abstract

In this talk, we discuss relativistic particles with zero spin from a different perspective. We intend to investigate the issue with an approach derived from Dunkl derivative. we considered (1+1)-dimensional DKP equation. we obtained eigenfunctions. Also we change the standard partial derivative for the Dunkl derivative. we solved the relativistic particle problem in a box according to the new formalism and found the energy spectrum for it. After that we investigate Scattering of potential step problem and Ramsauer-Townsend effect respectively and finally we will obtain the coefficients of transition and reflection.

Keywords: Dunkl drivative, DKP Equation, ν-deformed functions

QUANTUM GRAPHS AND THEIR VERTEX COUPLING CONDITIONS

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Abstract

In this talk, we briefly review the basic concepts of the quantum graph theory by introducing different coupling conditions at the graph vertices. In particular, we discuss the spectral properties of infinite periodic quantum graphs in the form of lattice graphs and loop arrays. Quantum graphs or more precisely metric graphs equipped with self-adjoint differential operators were introduced in 1953 by Ruedenberg et al. and three decades later rediscovered by Exner et al. Nowadays, quantum graphs have attracted a lot of attention; they arise as simplified models in different branches of science such as physics, mathematics, and chemistry; in particular, they have numerous applications in nanotechnology in the study of quantum chaos, photonic crystals, crystalline structures, graphene-related materials, and so on. A combinatorial graph Γ is a network-shaped structure consisting of a finite or infinite set of vertices and edge segments connecting the graph vertices. In order to make this purely combinatorial graph Γ a topological and metric structure, we introduce metric graphs; a graph Γ is called a metric graph if its each edge is identified with a real interval. A quantum graph is a metric graph equipped with a self-adjoint differential operator which acts on the wave function components on the graph edges and is accompanied by appropriate vertex conditions. The operator to investigate is the particle Hamiltonian, Schrodinger operator. In order to make the Hamiltonian operator a self-adjoint operator, one has to impose appropriate matching conditions to the boundary values of the functions at the graph vertices; this can be done in different ways, the most often considered cases are

- the general δ-coupling, in particular, Dirichlet and Kirchhoff conditions
- the general δ′-coupling, in particular, Neumann and anti-Kirchhoff conditions.
- a preferred-orientation coupling.

The talk is based on common works with Pavel Exner.

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BREATHERS SOLUTIONS IN TRAPPED TWO-COMPONENT CONDENSATES

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Abstract

We consider a 1D binary condensate trapped by a harmonic time-dependent potential and we solve analytically the associated coupled GrossPitaevskii equations using the Lax pair and Darboux transformation method. We find Akhmediev breather (AB), Ma soliton (MS) and rogue wave (RW) solutions. These solutions can be stabilized or destabilized depending on the trap frequency

BREATHERS TYPE SOLUTIONS OF COUPLED NONLINEAR SCHRÖDINGER EQUATION CNLS

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Abstract

By solving analytically the coupled nonlinear Schrödinger equation (CNLS) using Lax Pair and Darboux Transformation we start by broughting out general form of Breather (GB) solution and then we find that by changing the spectral parameters in the general solution we can derived other forms of solutions namely Akhmediev Breather, Ma Breather and Rogue wave.

Keywords: Lax pair, Darboux transformation, Breather, Akhmediev breather, Ma Breather, Rogue wave.

TEMPERATURE OF A ROTATING CHARGED BTZ BLACK HOLE WITH GENERALIZED and EXTENDED UNCERTAINTY PRINCIPLES (GEUP)

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Abstract

In this contribution, we will discuss the Hawking temperature of a rotating charged BTZ black hole in the combined formalism of extended and generalized uncertainty principles. To this end, at first we will introduce the extended and generalized uncertainty principles, and then, we will derive the modified Hawking temperature. Our result indicates that the modified Hawking temperature takes smaller values than the ordinary one.

MOLECULAR POTENTIALS FOR DESCRIPTION OF DIATOMIC MOLECULES AND APPLICATIONS TO QUANTUM TECHNOLOGY

C. EDET

EXACT SOLUTION OF ONE-DIMENSIONAL DIRAC OSCILLATOR IN SNYDER DE-SITTER MODEL

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Abstract

Using the momentum space representation, we present an exact solution of one-dimensional Fermionic oscillator for spin 1/2 particle with the Snyder De-Sitter (SDS) model, where the energy eigenvalue and eigenfunction are determined. The wave function is given in term of Gegenbauer polynomial. We also comment on the thermodynamic properties of the system.

1 Introduction

The relativistic harmonic oscillator (RHO) is one of the fundamental physics models, container of the bound states with a non-null residual energy, which is explained by the quantum confinement effect. In other words, this phenomenon proves the accuracy of quantum laws consequently. For example, in nuclear physics domain, RHO is the central potential of the nuclear shell model, and it has been used also as the confining two-body potential for quarks in particle physics. Since the article of Moshinsky et al., [1] Dirac relativistic oscillator raised a considerable attention by many researchers and has been studied intensively during the last several years [2]. Recently, at a microscopic scale of high energy, large amount of research work has been devoted to extend the study of quantum mechanics in the flat space Snyder model to curved space-time generalized algebra [3] in order to absorb the infinities vitiating the standard field theories.

The purpose of the present work is to solve the one-dimensional Dirac oscillator exactly in deformed space obeying the Snyder de-Sitter algebra which gives rise to the appearance of the minimal uncertainty in position as well as in momentum and also to see the effect of the deformation on the thermodynamics properties in relativistic systems with spin.

Let's start with the following commutation relation of deformed Heisenberg algebra in SDS model

$$[X_i, P_j] = i\hbar\left(\delta_{ij} + \alpha_1 X_i X_j + \alpha_2 P_i P_j + \sqrt{\alpha_1\alpha_2}\left(X_i P_j + P_j X_i\right)\right) \quad (1)$$

where α_1,α_2 are a small positive parameters of deformation, which gives rise to uncertainty Heisenberg relation

$$\Delta X_i \Delta P_j \geq \frac{1}{2}\left(\delta_{ij} + \gamma_{ij} + \alpha_1(\Delta X_i)^2 + \alpha_2\left(\Delta P_j\right)^2 - 2\sqrt{\alpha_1\alpha_2}\Delta X_i \Delta P_j\right), i = 1,2,3. \quad (2)$$

Where $\gamma_{ij} = \left(\sqrt{\alpha_1}\langle X_i\rangle + \sqrt{\alpha_2}\langle, P_j\rangle\right)^2 \geq 0$. Relation (2) implies the appearance of a nonzero minimal length in position and momentum uncertainties

$$(\Delta X)_{min} = \hbar\sqrt{\frac{\alpha_2(1+\gamma)}{1+2\sqrt{\alpha_1\alpha_2}}}, (\Delta P)_{min} = \hbar\sqrt{\frac{\alpha_1(1+\gamma)}{1+2\sqrt{\alpha_1\alpha_2}}} \quad (3)$$

X and P are realized in momentum space by

$$X_i = i\hbar\sqrt{1-\alpha_2 p^2}\partial_{p_i} + \tau\sqrt{\frac{\alpha_2}{\alpha_1}}\frac{p_i}{\sqrt{1-\alpha_2 p^2}} \quad , \quad (4)$$

$$P_i = -i\hbar\sqrt{\frac{\alpha_2}{\alpha_1}}\sqrt{1-\alpha_2 p^2}\partial_{p_i} + (1-\tau)\frac{p_i}{\sqrt{1-\alpha_2 p^2}} \quad . \quad (5)$$

Where p varies in the domain $\left]-\frac{1}{\sqrt{\alpha_2}}, \frac{1}{\sqrt{\alpha_2}}\right[$ and τ is an arbitrary real constant.

2 Dirac oscillator in one dimension

The stationary equation describing the Dirac oscillator in one dimension by the substitution $P \rightarrow P - i\beta m\omega X$ is given by [2]

$$[c\alpha(P - i\beta m\omega X) + \beta mc^2]\psi = E\psi, \quad (6)$$

where m is the rest mass, ω is the classical frequency of the oscillator and $\psi = \binom{f}{g}$ is a two-component spinor. Using the representation of Dirac matrices α and β [23], we obtain the following simultaneous equations:

$$c(-iP + m\omega X)g = (E - mc^2)f, \quad c(iP + m\omega X)f = (E + mc^2)g \quad (7)$$

In the momentum space realization of the position and momentum operators (4), (5), this system gives the following differential equation for the component f:

$$\left[-\frac{\alpha_1\gamma\gamma^*\hbar^2}{\alpha_2}(1-\alpha_2 p^2)\frac{\partial^2}{\partial p^2} + \left(\alpha_1\gamma\gamma^*\hbar^2 + \frac{2i\hbar\Omega}{\alpha_1\alpha_2}\right)p\frac{\partial}{\partial p} + \left(1 - \tau + \frac{\lambda\Omega}{\alpha_1^2} - m\omega\hbar\alpha_2\right)\frac{p^2}{(1-\alpha_2 p^2)} + \frac{2i\hbar\Omega}{\sqrt{\alpha_1\alpha_2}(1-\alpha_2 p^2)}\right]f(p) = \varepsilon f(p) \quad (8)$$

Where $\gamma = \left(1 + im\omega\sqrt{\frac{\alpha_2}{\alpha_1}}\right)$, $\Omega = (m^2\omega^2\alpha_2 + \alpha_1)\tau - \alpha_1$ and $\varepsilon = \frac{E^2 - m^2c^4}{c^2} + m\omega\hbar$.

Now, in order to solve Equation (8), we use the following transformations:

$$p \to \rho = \frac{1}{\kappa}\arcsin(\sqrt{\alpha_2}p), \quad \phi(\rho) = (1-q^2)^{\frac{1}{2}\left(\mu+\frac{\delta}{\kappa}\right)}f(q) \tag{9}$$

where $\kappa = \hbar\sqrt{\alpha_1\gamma\gamma^*}$, μ is a constant to be determined and q = sin (κ ρ) . Then, the equation becomes

$$\left[(1-q^2)\frac{\partial^2}{\partial q^2} - (2\mu+1)q\frac{\partial}{\partial q} + \frac{\epsilon}{\kappa^2} - \left(\mu+\frac{\delta}{\kappa}\right)\right]f(q) = 0. \tag{10}$$

With

$$\mu(\mu-1) = \frac{\delta}{\kappa}\left(\frac{\delta}{\kappa}+1\right) + \frac{\eta}{\kappa^2}, \delta = \frac{-i\hbar\Omega}{\kappa\alpha_1\alpha_2}, \eta = \frac{1-\tau}{\alpha_2} - m\omega\hbar + \left(\frac{\tau}{\alpha_1^2\alpha_2} + \frac{i\hbar}{\sqrt{\alpha_1\alpha_2}}\right)\Omega$$

and

$$\epsilon = \varepsilon - \frac{i\hbar\Omega}{\sqrt{\alpha_1\alpha_2}}.$$

At this point, in order to avoid complex eigenvalues ϵ of the differential Equation (10), we must impose a condition to eliminate the imaginary term by setting Ω = 0 in Equation (10) which fixes the value of the arbitrary parameter τ as $\tau = \alpha_1(m^2\omega^2\alpha_2+\alpha_1)^{-1}$. This brings the differential equation in the Gegenbauer form[4].

$$\left[(1-q^2)\frac{\partial^2}{\partial q^2} - (2\mu+1)q\frac{\partial}{\partial q} + n(n+2\mu)\right]f(q) = 0. \tag{11}$$

where n and μ (a non-negative integer) satisfy

$$\begin{cases} \frac{\epsilon}{\kappa^2} - \mu = n(n+2\mu) \\ \mu = \frac{1}{2}\left(1+\sqrt{1+\frac{4}{\kappa^2}\left(\frac{m^2\omega^2}{m^2\omega^2\alpha_2+\alpha_1} - m\omega\hbar\right)}\right) \end{cases} \tag{12}$$

Now, the solution of Equation (10) can be expressed in terms of Gegenbauer polynomials

$$\psi(p) = \begin{pmatrix} f \\ g \end{pmatrix} = \Lambda(1-\alpha_2p^2)^{\frac{\mu}{2}}\left[M(p)C_n^{\mu}(\sqrt{\alpha_2 p}) + N(p)C_{n-1}^{\mu+1}(\sqrt{\alpha_2 p})\right] \tag{13}$$

Where $M(p) = \begin{pmatrix} 1 \\ \frac{ic[(1-\tau\gamma)+i\hbar(\mu\gamma\sqrt{\alpha_1\alpha_2})]p}{E+mc^2} \end{pmatrix}$, $N(p) = \begin{pmatrix} 0 \\ \frac{i\hbar\mu\gamma\sqrt{\alpha_1}\sqrt{1-\alpha_2p^2}}{E+mc^2} \end{pmatrix}$

with Λ is a normalization constant. Employing the abbreviations of Equations (10) with the first relation of (12), it is straightforward to show that the deformed Dirac oscillator energy spectrum E_n is

$$E_n = \pm mc^2\sqrt{1+\frac{2\omega\hbar n}{mc^2} + \frac{\hbar^2(m^2\omega^2\alpha_2+\alpha_1)n^2}{m^2c^2}}. \tag{14}$$

It is remarkable that the energy spectrum expression of the considered system contains an additional correction term which depends on the deformation parameters α_1 and α_2, and its deviation grows quickly with n. It should be emphasized that this result explains confinement at the high energy area.

3 Thermal properties

In order to obtain the thermodynamic properties of the deformed Dirac oscillator with Snyder de Sitter commutation relations at finite temperature T, we consider the partition function of such a system according to the Boltzmann factor K_B as follows:

$$Z = \sum_{n=0}^{\infty} \exp\left(\frac{-E_n}{K_B T}\right) = \sum_{n=0}^{\infty} exp\left(-\frac{mc^2}{K_B T}\sqrt{\gamma_1 + \gamma_2 n + \gamma_3 n^2}\right). \quad (15)$$

Here the expression of the other parameters γ_1, γ_2 and γ_3 we can obtain them from the spectrum expression. Now, following Euler Maclaurin's formula and neglecting second-order contributions in λ a simple calculation, one can reduce the last form of the partition function Z:

$$Z \simeq \frac{(K_B T)^2}{\hbar\omega mc^2}\left(1 - (K_B T)^2\lambda\right) \text{ wehre } \lambda = \frac{1}{c^2}\left(\frac{\alpha_1}{m\omega} + \alpha_2\right). \quad (16)$$

At this stage, according to the definitions mentioned below, we can obtain the thermodynamic properties of our physical system, such as free energy F, mean energy U, specific heat C, and entropy S, as follows:

$$F = -K_B T lnZ \simeq -K_B T ln\left(\frac{(K_B T)^2}{\hbar\omega mc^2}\left(1 - 3\lambda(K_B T)^2\right)\right), \quad (17)$$

$$U = K_B T^2 \frac{\partial lnZ}{\partial T} \simeq 4K_B T\left(1 - \frac{1}{2(1 - 3\lambda(K_B T)^2)}\right), \quad (18)$$

$$C = \frac{\partial U}{\partial T} \simeq 4K_B\left(1 - \frac{1 + 3\lambda(K_B T)^2}{2(1 - 3\lambda(K_B T)^2)^2}\right), \quad (19)$$

$$S = -\frac{\partial F}{\partial T} \simeq K_B\left[\frac{2 - 12\lambda(K_B T)^2}{1 - 3\lambda(K_B T)^2} + ln\left(\frac{(K_B T)^2}{\hbar\omega mc^2}\left(1 - 3\lambda(K_B T)^2\right)\right)\right]. \quad (20)$$

In what follows, all these results of thermodynamic properties of the deformed Dirac oscillator are numerically shown below in Figures 1-4, as a function of dimensionless temperature variable T for different values of the deformation parameter λ, that is, λ = 0, 0.1, and 0.9, where we have used ($\hbar = K_B = \omega = m = c = 1$).

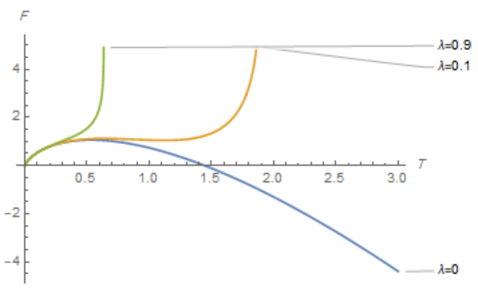


Fig1: The free energy F for different values of λ

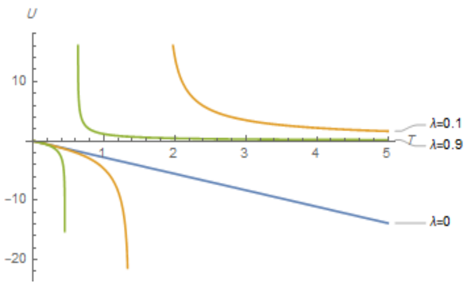
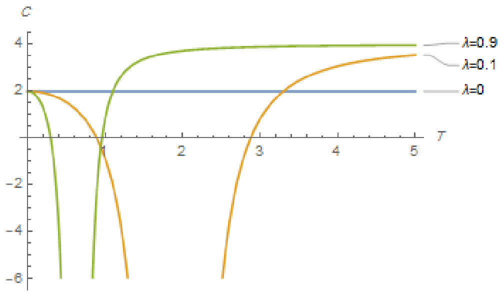


Fig2: The mean energy U for different values of λ.



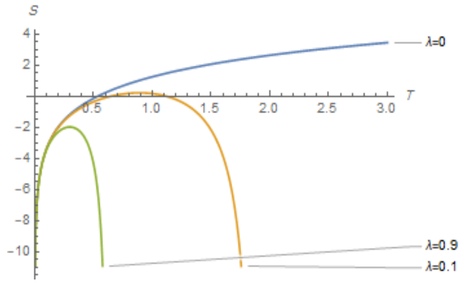


Fig3: The heat capacity C for different values of λ **Fig4**: The entropy S for different values of λ

According to the factor $(1 - 3\lambda(K_B T)^2)$ which is present in the thermodynamic formulas, we clearly see the appearance of a critical point for the temperature value $T_c = \frac{1}{\sqrt{3\lambda}}$ which depends purely on the deformation parameter λ.

4 Conclusions

We study an explicit calculation of the one dimensional deformed Dirac oscillator in momentum space with Snyder-de Sitter commutation relations which lead to a non-zero minimal uncertainty in the measurement of the position as well as of the momentum. The exact solution is obtained where the wave functions are expressed in terms of Gegenbauer polynomials. The energy spectrum of the system is deduced with an additional correction, which depends on the deformation parameters α_1 and α_2, and its deviation grows quickly with n which can be related with confinement. Subsequently, in the regime of high temperatures, we show that the thermodynamic properties of our system have also been influenced by this deformation of space.

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QUANTUM ENTANGLEMENT: THE BELL-CHSH INEQUALITIES

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Abstract

The separability problem is one of the basic and emergent problems in the present and future quantum information processing. In this contribution, we review the first operational criterion of separability, namely the Bell-CHSH inequalities, providing a good starting point for reading about the topic and directing the interested reader to more in-depth resources.

Keywords: Quantum entanglement; Separability criterion; Bell's inequalities.

1 Introduction

Albert Einstein was skeptical of quantum mechanics, particularly its copenhagen interpretation [1]. In the May 15 1935 issue of Physical Review, Albert Einstein co-authored a paper with Boris Podolsky and Nathan Rosen who were his two postdoctoral research associates at the Institute for Advanced Study. The article was entitled *Can quantum mechanical description of physical reality be considered complete*? [2]. In this study, the three scientists proposed a thought experiment known today as EPR paradox that attempted to show that the quantum mechanical description of physical reality given by wave functions is not complete.

However, Einstein, Podolsky and Rosen did not coin the word entanglement. Erwin Schrödinger in his correspondence with Einstein, following the EPR paper used the word Verschränking (in German) translated by himself in English as entanglement, to describe the correlations between two particles that interact and then separate as in the EPR thought experiment. He shortly thereafter published a seminal paper defining and discussing the notion of entanglement [3]. In this seminal paper, Schrödinger recognized the importance of the concept, and stated : " I would not call (entanglement) one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought ". Einstein was disturbed by the theoretical concept of quantum entanglement, which he called *Spooky action at distance*. Einstein did not believe two particles could remain connected to each other over great distances: doing so, he said, would require them to communicate faster than the speed of light, something he had previously shown to be impossible. Like Einstein, Schrödinger was dissastified with the concept of entanglement, because it seemed to violate the speed limit on the transmission of information implicit in the theory of relativity. The EPR paper [2] generated significant interest among physicists and quickly became a centerpiece in debates over the interpretation of quantum theory, debates that continue today.

The question of expected locality of the entangled quantum systems raised by EPR allowed John Stewart Bell to discover his famous inequalities serving as a test and demonstration of strange properties of the simplest entangled wave function represented by a singlet state [4,5]. Still one had to wait long for the proposals and pratical applications of quantum entanglement. Until 1975, a decisive experiment based on the violation of Bell's inequalities and verifying the veracity of quantum entanglement was missing. The experiment led by French physicist Alain Aspect at the Ecole Supérieure d'optique in Orsay between 1980 and 1982 was the first quantum mechanics experiment to demonstrate the violation of Bell's inequalities [6, 7]. This experiment is called the Aspect's experiment. It confirmed the predictions of quantum mechanics and thus confirmed its incompabilities with local theories. Quantum entanglement is a phenomenon which has no counterpart in classical physics. It can be seen as the most non-classical feature of quantum mechanics that has risen numerous philosophical, physical and mathematical questions since the early days of the quantum theory.

The fundamental question in quantum entanglement theory is which states are entangled and which states are not and this question is still an open problem today, both from the theoretical and experimental point of view, and known as the separability problem, that has been solved for pure states [8], and for 2×2 and 2×3 systems [9]. A separability condition can be necessary or necessary and sufficient conditions for separability. A necessary condition for separability has to be fulfilled by every separable state. In that case, if a state does not fulfill the condition, it has to be entangled, but if it fulfills we cannot conclude. On the other hand, a necessary and sufficient condition for separability can only be satisfied by separable states, if a state fulfills a necessary and sufficient condition, then we can be sure that the state is separable.

2 Bell-CHSH inequalities

An operational criterion of separability is a recipe that can be applied to an explicit density matrix ρ, giving some immediate answer like " ρ is entangled" or "ρ is separable" or this "criterion is not strong enough to decide whether ρ is separable or entangled ".

The Bell inequality is the first example of operational criterion of separability. It was originally designated to test predictions of quantum mechanics against those of a local hidden variables theory [4]. Bell's inequalities were initially dealing with two qubits, i.e two-level systems and provide a necessary criterion for the separability of 2-qubits states. For pure states, Bell's inequalities are also sufficient for separability. It has been proven by Gisin that any non-product state of two-particle systems violates a Bell-inequality [10]. This inequality which involves three vectors in a real space $\mathbb{R}^3$ determing which component of a spin to be measured by each party or three, has been extended for the case involving four vectors by Clauser, Horne, Shimony and Holt (CHSH) in 1969 [11]. The Bell-CHSH inequality also provides a test to distinguish entangled from non-entangled states.

Consider a system of two qubits. Let A and A' denote observables on the first qubit, B and B' denote observables on the second qubit, the Bell-CHSH inequality says that for non-entangled states, means for states of the form $\rho = \rho_1 \otimes \rho_2$, or mixtures of such states, the following inequality holds:

$$|\langle\, A \otimes B + A \otimes B' + A' \otimes B - A' \otimes B' \,\rangle_\rho| \leq 2, \tag{1}$$

where

$$\langle A \otimes B \rangle_\rho := \mathrm{Tr}\ \rho(A \otimes B);\ \langle A \otimes B \rangle_\psi = \langle \psi | A \otimes B | \psi \rangle \tag{2}$$

for the expectation value of $A \otimes B$ in the mixed states ρ or pure state $|\psi\rangle$. As an example, we consider a two qubits state $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, and the observables

$$A = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z),\ A' = \frac{1}{\sqrt{2}}(\sigma_x - \sigma_z),\ B = \sigma_x,\ B' = \sigma_z, \tag{3}$$

where σ_x and σ_z are Pauli matrices. We have then explicitly

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \tag{4}$$

and

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; \quad A' = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}; \tag{5}$$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad B' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{6}$$

It is easy to check that

$$\langle\phi|\ A \otimes B + A \otimes B' + A' \otimes B - A' \otimes B'\ |\phi\rangle = 2\sqrt{2}. \tag{7}$$

The state $|\phi\rangle$ which violates the Bell-CHSH inequality is a well known entangled state, and is one of the Bell pairs, maximally entangled state. The maximal violation of (1), for entangled states follows from an inequality of Cirelson [12]

$$|\langle A \otimes B + A \otimes B' + A' \otimes B - A' \otimes B' \rangle_\rho| \leq 2\sqrt{2}. \tag{8}$$

The equality in equation (8) can be attained by the singlet state.

3 Conclusion

Historically, Bell-CHSH inequalities were the first tool for the recognition of entanglement; however, it is well-known for some time that the violation of a Bell-CHSH inequality is only a sufficient condition for entanglement and not a necessary one, and that there are in fact many entangled states that satisfy them [13]. The Bell-CHSH inequalities were generalized to N qubits, whose violations provide a criterion to distinguish the totally separable states from the entangled states [14–16]. The Bell's inequality is only a mathematical theorem and the relation between Bell's inequalities and convex geometry is also well-known [17].

Acknowledgment

This talk is based on a recent work co-authored by Ms. Honorine GNONFIN cited as arXiv: 2208.04747[quant-ph]

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SOLUTION OF DKP EQUATION WITH GENERALIZED EXTENDED MOMENTUM OPERATOR

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Abstract

In this work, we present the solution of DKP equation in the context of generalized momentum operator p, when we choose the function f(x) is linear in the representation of p and also the scalar and Vector potentials (S(x), V (x)) are linear, this solution lead to calculate the energy Spectrum En and the wave functions φn (x) which are given by the confluent Heun differential equation HC (a, b, c, d, e, y), the limits cases are deduced

Keywords: Duffin-Kemmer-Petiau Equation, Generalized Extended Momentum Operator.

GUP-CORRECTED THERMAL PROPERTIES OF ROTATING CHARGED BTZ BLACK HOLE IN GRAVITY'S RAINBOW

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Abstract

In this contribution, we will talk on the thermal properties of a rotating charged BTZ black hole in gravity's rainbow in the generalized uncertainty principle framework. To this end, we will briefly introduce the GUP and gravity's rainbow formalisms and then we will derive the GUP-GR corrected Hawking temperature, entropy, Helmholtz free energy, internal energy and heat capacity functions. Our findings show that there is a physical and nonphysical region associated with the stability of the black hole.

Please note that this contribution is based on our published paper, in Int. J. Mod. Phys. A 37, 2250126 (2022).

THE INFLUENCE OF BOTH NONCOMMUTATIVITY AND NON-LOCALITY ON THE CONTINUITY EQUATION

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We present the continuity equation in the presence of local and non-local potentials that arising from electron-electron interaction within both commutative and noncommutative phase-spaces. As an application about non-locality, we consider the Frahn-Lemmer potential, thereafter we examine the effect of the noncommutativity on both the locality and non-locality. Properly, the definition of current density in commutative case cannot satisfy the condition of current conservation, so that to solve this problem, we give a new definition of the current density including the contribution due to the non-local potential. We show that the calculated current based on the new definition of the current density maintains the current. On the other hand, for the case of noncommutative framework, the conservation of the current density is completely violated; and the noncommutativity is not suitable for describing the current density in presence of non-local and local potentials. However, under some conditions, we also modify the current density to solve this problem. Taking into account that the used noncommutativity is encoded through both the Bopp-shift and Moyal-Weyl product.

DUNKL-ELECTROSTATICS AND DUNKL-MAXWELL EQUATION

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Abstract

Dunkl formalism is been built by substituting the ordinary derivative with a combined difference-differential derivative. The latter form let us derive reflection symmetric and antisymmetric solutions at the same time. This fact can be regarded as a parity-dependent solution. Recently, Dunkl formalism are being used in relativistic and non-relativistic quantum mechanics. However, more is possible. In this talk, we will introduce the usage of Dunkl-formalism in electrostatics. Moreover, we will construct the Dunkl-Maxwell equations.

GENERALIZED-DUNKL HARMONIC AND ANHARMONIC OSCILLATORS

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Abstract

Recent studies shows that the Dunkl formalism can be used to examine relativistic and nonrelativistic particle's dynamics according to the parity. Generally, the Dunk derivative is given with one free parameter in one dimension. However, the most general form can be given with three parameters. The latter form can produce a deeper analysis and a better fit the experimental results. In this talk, we take into account the most general form to re-examine harmonic and anharmonic oscillator solutions. This talk is a part of our published paper in Phys. Scr. **97**, 125305 (2022).

ANALYTICAL SOLUTION OF A NON-CENTRAL POTENTIAL PLUS PSEUDO-HARMONIC OSCILLATOR POTENTIAL

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Abstract

In this work, we study the wave equations in 3D Euclidian space for a non-central potential consisting of a Kratzer term , harmonic oscillator and a Hautot term

$$V(r,\theta) = Kr^2 + D_r r^{-2} + \frac{\hbar^2}{2\mu^2}(\alpha cos^2\theta + \beta\cos\theta + \gamma)\sin\theta^{-2} r^{-2}.$$

For Schrödinger equation, we obtain the analytical expressions of the energies and the wave functions of the system. We also study a system of particule in Ring-shaped potential plus pseudo-harmonic oscillator as a limit of $V(r,\theta)$ when $\alpha = \beta = 0$ the dependence of energies on the parameters D_r and γ. We find that the D_ term tends to dissociate the system, and thus counteracts the Coulomb binding effect, and that the Dr term can either amplify or decrease this effect according to its sign.

Keywords: Schrödinger equation, non-central potential, Kratzer potential, Ring-shaped potential, harmonic oscillator

1- Introduction

The non-central potentials especially for which the Schrödinger equation can be solved exactly by separation of variables have been found many applications, particularly in quantum chemistry [5]. They are used to describe the quantum dynamics of ring-shaped molecules like benzene molecule, they have solved the 3 dimension Schrödinger equation by using the Kustaanheimo-Stiefel transformation, another application of non-central potential is the interactions between deformed nuclei pairs [6] . The potentials without spherical symmetry have some applications within the nanostructure theory [7], and also help us about structuring the metallic glasses [8]. The non-central potentials serve to the theory of the material sciences, for example, describing microscopic elasticity, and obtaining of elastic constants of a cubic crystal [9]

We solve the Schrödinger equation of a particule moving in non-central potential plus Kratzer potential and harmonic oscillator, we found the energy and the wave function finally we focused to the Ring-shaped potential plus Kratzer potential when we plotted the energy in terms of radial momentum and Ring-shaped parameter and we discussed the results.

2- 3D Schrödinger Equation

The Schrödinger equation of the system is written as

$$\left[\frac{-\hbar^2}{2\mu}\Delta + V(r,\theta)\right]\psi = E\psi \tag{1}$$

$$\left[\frac{-\hbar^2}{2\mu}\Delta + \mu\left(V(r) + \frac{\hbar^2}{2\mu^2}\frac{\alpha cos^2\theta + \beta\cos\theta + \gamma}{r^2 sin^2\theta}\right)\right]\psi = E\psi \quad (2)$$

To separate the variables, it is better the using of the spherical coordinates (r, θ, φ) then the Schrödinger equation is written as

$$\left[\frac{\partial}{\partial r}^2 + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2\mu^2}{\hbar^2}V(r) + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} - \frac{1}{sin^2\theta}\frac{\partial}{\partial\varphi} - \frac{\alpha cos^2\theta + \beta\cos\theta + \gamma}{sin^2\theta}\right)\right]\psi(r,\theta) = -\frac{2\mu E}{\hbar}\psi(r,\theta) \quad (3)$$

The variables can be separated when the wave function is written as: $\psi(r,\theta) = \exp(im\varphi)R(r)\Theta(\theta)$

So, this gives as two equations the radial equation and the angular one

$$\frac{d^2\Theta(\theta)}{d\theta^2} + \cot\theta\frac{d\Theta(\theta)}{d\theta} - \frac{m^2}{sin^2\theta}\Theta(\theta) - \frac{\alpha cos^2\theta + \beta\cos\theta + \gamma}{sin^2\theta}\Theta(\theta) - E_\theta\Theta(\theta) = 0 \quad (4)$$

$$\frac{d^2R(r)}{dr^2} + \frac{2}{r}\frac{dR(r)}{dr} + \left[\frac{2\mu}{\hbar^2}E - \left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)\frac{1}{r^2} - \frac{2\mu^2 k}{\hbar^2}r^2\right]R(r) = 0 \quad (5)$$

We have to solve the angular equation (4) to find the constants E_θ and then we use these angular eigenvalues to solve the radial equation (5), this will give us the energies E of the system and also the wave function $\psi(r,\theta)$

3- Solution of Angular Equation

After some substitutions we get a hypergeometric equation and the solution is hypergeometric function

$$\Theta(\theta) = N_\theta \cos\theta^{2\rho}\left(\frac{\theta}{2}\right)\left(1 - \cos\left(\frac{\theta}{2}\right)\right)^\sigma$$

$$F\left(-l, l + 1 + (l^2 + \alpha - \beta + \gamma)^{1/2} + +(l^2 + \alpha + \beta + \gamma)^{1/2}; 1 + +(l^2 + \alpha - \beta + \gamma)^{1/2}; \cos\theta^2\left(\frac{\theta}{2}\right)\right) \quad (7)$$

And the constant of separation is

$$E_\theta = \frac{1}{4} + \alpha - \left[l + \frac{1}{2}(m^2 + \alpha - \beta + \gamma)^{1/2} + \frac{1}{2}(m^2 + \alpha + \beta + \gamma)^{1/2} + \frac{1}{2}\right]^2 \quad (8)$$

Were the constants being

$$\rho = \frac{1}{2}\left(l^2 + \alpha - \beta + \gamma\right)^{1/2} \quad (9)$$

$$\sigma = \frac{1}{2}\left(l^2 + \alpha + \beta + \gamma\right)^{1/2} \quad (10)$$

4- Solution of Radial Equation

After some changes of variables, the radial equation becomes is the Kummers (conuent hypergeometric) di⁄erential equation and the solution of this equation that is regular at $r = 0$ or $\rho = 0$is the degenerate hypergeometric function or the Kummers function:

$$R(r) = N_r(r)^{-\frac{1}{2}+\frac{1}{2}\sqrt{1+4\left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)}} e^{-r^2\sqrt{\frac{\mu^2 k}{2\hbar^2}}}$$

$${}_1F_1\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)} - 4\sqrt{\frac{\mu}{k}}E, 1 + \sqrt{\frac{1}{4} + \left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)}, 2r^2\sqrt{\frac{\mu^2 k}{2\hbar^2}}\right) \quad (11)$$

And the energy is $\alpha^2 = \frac{2\mu}{\hbar^2}E$, $R(r) = r^\beta e^{-\lambda r^2} f(r)$,$\rho = 2\lambda r^2$

$$\rho\frac{d^2 f(\rho)}{d\rho^2} + \left(\frac{(2\beta+3)}{2} - \rho\right)\frac{df(\rho)}{d\rho} - \left(\frac{1}{4}(2\beta+3) - \frac{1}{8}\sqrt{\frac{2\hbar^2}{\mu^2 k}}\alpha^2\right)f(\rho) = 0$$

$$f(\rho) = N_r\ {}_1F_1\left(\frac{1}{4}(2\beta+3) - \frac{1}{8}\sqrt{\frac{2\hbar^2}{\mu^2 k}}\alpha^2, \frac{(2\beta+3)}{2}, \rho\right)$$

$$E_r = \hbar\sqrt{2k}\left(2n_r + 1 + \sqrt{\frac{1}{4} + \left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)}\right) \quad (12)$$

5- Energy and Wave function of the System

We deduce the wave function of our system $\psi(r,\theta) = \exp(im\varphi)R(r)\Theta(\theta)$ from the angular (7) part and the radial part (11)

$$\psi(r,\theta) = N\ \exp(im\varphi)\,(r)^{-\frac{1}{2}+\frac{1}{2}\sqrt{1+4\left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)}} e^{-r^2\sqrt{\frac{\mu^2 k}{2\hbar^2}}}$$

$${}_1F_1\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)} - 4\sqrt{\frac{\mu}{k}}E, 1 + \sqrt{\frac{1}{4} + \left(\frac{2\mu^2}{\hbar^2}D_r - E_\theta\right)}, 2r^2\sqrt{\frac{\mu^2 k}{2\hbar^2}}\right)$$

$$\cos\theta^{2\rho}\left(\frac{\theta}{2}\right)\left(1 - \cos\left(\frac{\theta}{2}\right)\right)^\sigma$$

$$F\left(-l, l+1+(l^2+\alpha-\beta+\gamma)^{1/2} + +(l^2+\alpha+\beta+\gamma)^{1/2}; 1 + +(l^2+\alpha-\beta+\gamma)^{1/2}; \cos\theta^2\left(\frac{\theta}{2}\right)\right) \quad (13)$$

We substitute the constant of separation (8) in the expression of energy (12), we find the final expression of energy as

$$E = \hbar\sqrt{2k}\left(2n_r + 1 + \sqrt{\frac{2\mu^2}{\hbar^2}D_r - \alpha + \left[l + \frac{1}{2}(m^2+\alpha-\beta+\gamma)^{1/2} + \frac{1}{2}(m^2+\alpha+\beta+\gamma)^{1/2} + \frac{1}{2}\right]^2}\right) \quad (14)$$

$$n_r = 0,1,2,\ldots, l = 0,1,2,\ldots, m = 0,\pm1,\pm2,\ldots,$$

We can study in this case the limit at $\alpha = \beta = 0$ and $k = \frac{1}{2}\omega$ where the potential is the pseudo-harmonic ring-shaped potential $V(r,\theta) = \frac{1}{2}\omega\, r^2 + D_r r^{-2} + \frac{\hbar^2}{2\mu^2}\frac{\gamma}{r^2 sin^2\theta}$,this potential has an application field in quantum chemistry as a model for the Benzene molecule

$$E_{PHO+Rs} = \hbar\omega\left(2n_r + 1 + \sqrt{\frac{2\mu^2}{\hbar^2}D_r + \left[l + (m^2+\gamma)^{1/2} + \frac{1}{2}\right]^2}\right) \quad (15)$$

If we take the limit of harmonic oscillator is deduced where $D_r = 0,\ \gamma = 0$ and compare it by the energy of Comparing to the energy of 3D harmonic oscillator we find

$$2n_r + l + m = n \Rightarrow 2n_r = n - l - m$$

So, the energy can be written as

$$E_{PHO+Rs} = \hbar\omega\left(n - l - m + 1 + \sqrt{\frac{2\mu^2}{\hbar^2}D_r + \left[l + (m^2+\gamma)^{1/2} + \frac{1}{2}\right]^2}\right) \quad (16)$$

$$n = 0,1,2,\ldots, l = 0,1,2,\ldots, m = 0,\pm1,\pm2,\ldots,$$

We noted that the expression under the root is always positive that means we haven't a critical value for γ and D_r

By using the Hartree units the last equation becomes

$$E_{PHO+Rs} = \omega\left(n - l - m + 1 + \sqrt{2D_r + \left[l + (m^2+\gamma)^{1/2} + \frac{1}{2}\right]^2}\right) \quad (17)$$

We plotted the variation of this energy in terms of γ the parameter of the ring-shaped potential and for different values of radial momentum D_r (Figures 1)

From the radial equation we can plotted the effective potential, for the ring-shaped potential was showing in Figures2;:::; 5) ;we note that the state of ring shaped potential are bounded Whatever the energy level, and it is not affected by the radial momentum D_r or the parameter of ring-shaped potential

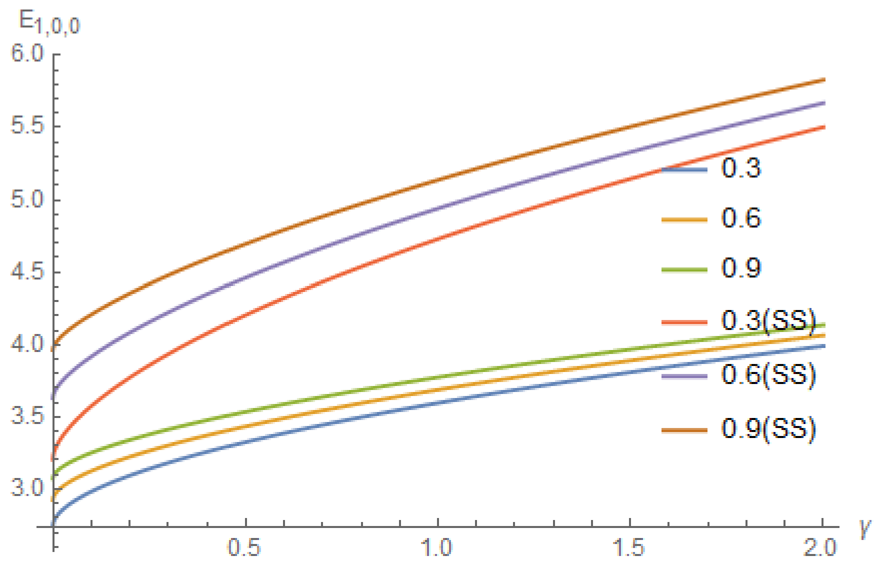

Figure 1: The energy $E_{1,0,0}$ $(n = 1, l = 0, m = 0, \omega = 1)$ of PHO+ring shaped potential for terms of γ and $D_r = 0.3,0.6\ and 0.9$

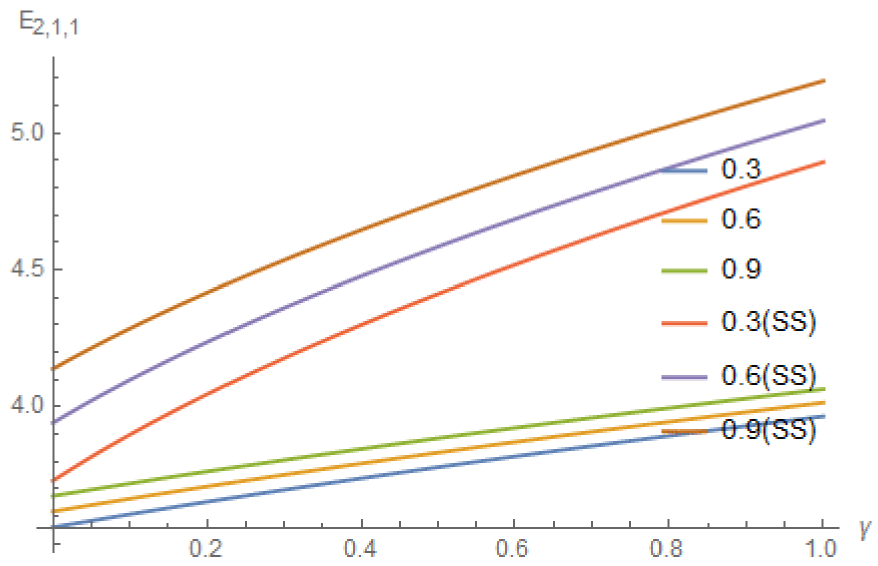

Figure 2: The energy $E_{2,1,1}$ $(n = 2, l = 1, m = 1, \omega = 1)$ of PHO+ring shaped potential for terms of γ and $D_r = 0.3,0.6\ and 0.9$

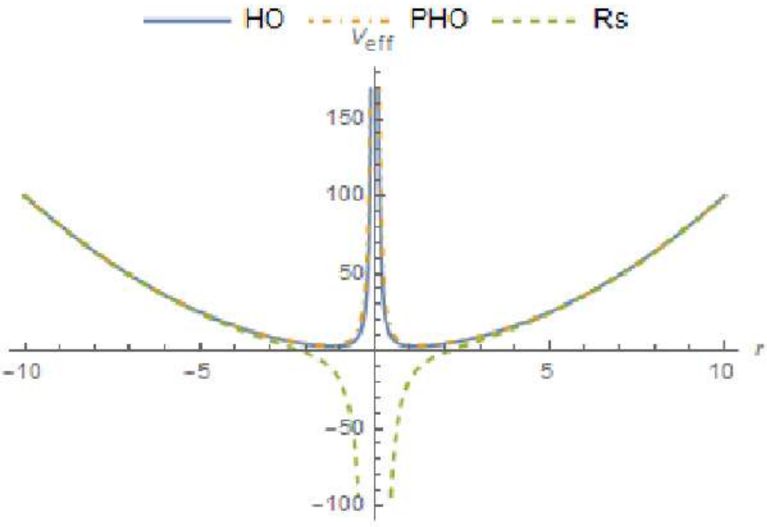

Figure 3: Veff the effective potential of HO, PHO and PHO +ring-shaped for l = 1;m = 1;D_r = 1; $\gamma = 10$ in terms of r

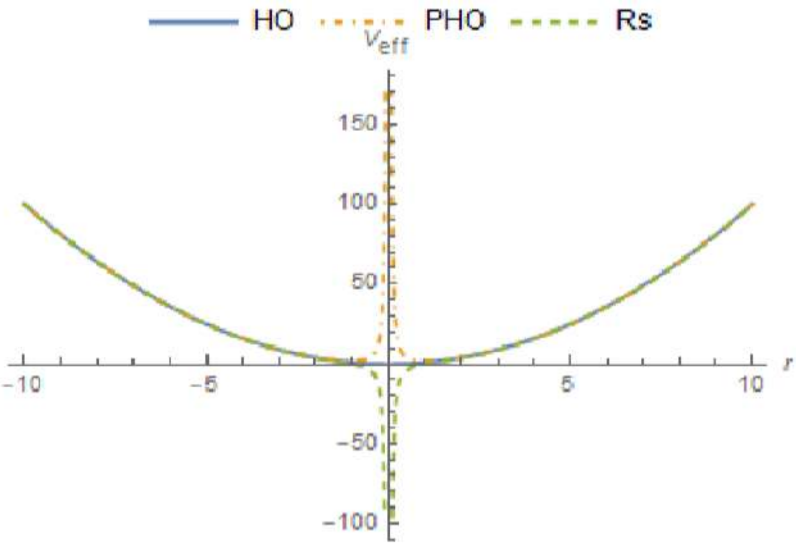

Figure 4: Veff the effective potential of HO, PHO and PHO +ring-shaped for l = 1; m = 1;D_r = 10; $\gamma = 1$ in terms of r

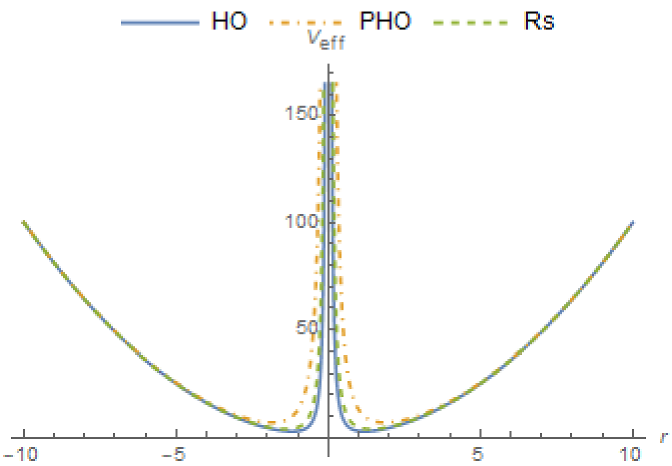

Figure 5: Veff the effective potential of Colombian kratzer and kratzer +ring-shaped for
$l = 2, m = 1, D_r = 1$ and $\gamma = 1$ in terms of r

6- Conclusion

The non-central potentials remove the degeneracy occurrence of the three quantum numbers (n, l, m). All the states of energy are bound state whatever its level or its radial momentum and these states don't affect by the parameter of the ring-shaped potential

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INVESTIGATION OF NONLINEAR KLEIN-GORDON DIFFERENTIAL EQUATION BASED ON GUP FORMALISM BY ADOMIAN DECOMPOSITION METHOD

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Abstract

In this research, we investigate the Lagrangian of Klein-Gordon field with forth order interaction and a breaking symmetry term. The Lagrangian in the presence of generalized uncertainty principle (GUP) formalism is written and according to Euler-Lagrange equation, the fourth-order nonlinear differential equation is found. We try to find a suitable solution.

Keywords: Nonlinear Klein- Gordon equation; Generalized Uncertainty Principle (GUP); Lagrangian.

FERMİ ENERGY İN A -DEFORMED FRAMEWORK

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Abstract

In this article, the κ-deformation formalism is investigated. Using the κ-deformation, the Fermi energy, for box problem with the Schrödinger equation, has been investigated and calculated. In addition, we calculated the internal energy and pressure, and also calculated the radius of the white dwarf with the pressure due to degeneracy and gravity.

SPECTRAL PROPERTIES of DOMAINS WITH SMALL RESONATORS

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Let Ω be a domain in $\mathbb{R}^n$, and let $\Omega_\varepsilon = \Omega \setminus \cup_{k=1}^m S_{k,\varepsilon}$, where $\varepsilon > 0$ is small parameter, $S_{k,\varepsilon}$ are closed surfaces with small suitably scaled "windows" through which the bounded domains enclosed by these surfaces ("resonators") are connected to the outer domain. The resonators shrink to points as $\varepsilon \to 0$. We demonstrate that in the limit $\varepsilon \to 0$ the spectrum of the Laplacian on Ω_ε with the Neumann boundary conditions on $S_{k,\varepsilon}$ and the Dirichlet boundary conditions on the $\partial\Omega$ converges (in the Hausdorff-type metrics) to the union of the spectrum of the Dirichlet Laplacian on Ω and the numbers γ_k, $k = 1, \ldots, m$; γ_k equals $1/4$ times the limit of the ratio between the capacity of the kth window and the volume of the kth resonator. Also, an application of this result is presented: we construct an unbounded waveguide-like domain with resonators such that the eigenvalues of the Laplacian on this domain lying below the essential spectrum threshold do coincide with prescribed numbers.

Bose mixtures beyond the mean field

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Abstract

The main goal of this work is to account for many body effects in ultra cold binary mixtures. This is performed by a consistent variational approach, free of Perturbative hypothesis. The intra and interspecies correlations appear on the same footing. This allows us to identify regimes where the latter may play dominant roles in the dynamics of the mixture. Furthermore, by considering short and long range potentials, we can also determine the relative weights of each type of correlations.

The fundamental idea is to work with a density operator containing both quantum and thermal effects. By a judicious choice of the representation, this operator will allow for a unique definition of the Hilbert state for the mixture. The Hamiltonian is chosen as a two body operator as one knows from experiments that higher order interactions play only minor roles in ultra cold ultra dilute gases. By using a variational principle, one obtains a set of dynamical equations governing the various densities (condensed, non condensed and anomalous) as well as the various one body correlations (local and non local, intra and interspecies). The system of partial differential equations has to be solved numerically. One may consider the equilibrium equations, the output of which can be directly compared to experiments. Homogeneous and trapped case are also considered.

Keywords: Ultra Cold gases, Mixtures, Gross-Pitaevskii, TDHFB, Finite Temperature affects, LHY, Post-Gaussian Approximation, Correlation functions, Triplets.

Bound states in soft quantum waveguides and layers

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Abstract

We consider Schrödinger operator with confining potentials depending on the distance to the infinite planar curve or spatial surface. Or main goal is to show the existence of discrete spectrum below the essential spectrum threshold. The overview of known results is presented.

GAZEAU-KLAUDER COHORENT STATES FOR POSITION-DEPENDENT MASS

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In this paper, we construct coherent states à la Gazeau-Klauder for a particle with position-dependent mass trapped in an infinite square well. We show that these states satisfy the Klauder's mathematical condition to build coherent states. We compute and analyse some statistical propertiesof these states. We find that these states are sub-Poissonian in nature.

Keywords: Gazeau-Klauder states; Position dependent mass

1 Introduction

La dynamic of systems with position-dependent mass (PDM) have found in the last decades applications in the fields of material science [1–3] and condensed matter physics [4, 5]. Recently, the study of free PDM system in strong quantum gravitational background fields have been realized [6]. It has been shown that,by increasing the quantum gravitational effect, the PDM of the particle increases and induces deformations of the quantum energy levels. These deformations are more pronounced as one increases the quantum levels allowing, the particle to jump from one state to another with low energies and with high probability densities. In the present paper, we construct the Gazeau-Klauder coherent states [7] for the spectrum of this position deformed mass system [6]. We show that these states satisfy the Klauder's mathematical conditions to build coherent states [8, 9]. We also explore the statistical properties [10] of these states, such as the photon distribution, the photon mean number, the intensity correlation and the Mandel parameter. We find that these states are sub-Poissonian in nature.

The paper is organised as follows: In the next section, we review in the dynamics of PDM in square well potential [6]. In section 3, we construct GK [7] coherent states for the deformed-spectrum of PDM [6]. Finally, we discuss the quantum statistical properties of the constructed coherent states and we show that theses have sub-Poisson statistics

2 Position-dependent mass Schrödinger equation

The most general Hamiltonian for a particle with position-dependent mass (PDM) $m(\hat{x})$ can be writen in the form [11]

$$\hat{H} = \frac{1}{4}\left[m^{\alpha}(\hat{x})\hat{p}m^{\beta}\hat{p}m^{\gamma}(\hat{x}) + m^{\gamma}(\hat{x})\hat{p}m^{\beta}\hat{p}m^{\alpha}(\hat{x})\right] + V(\hat{x}) \tag{1}$$

where α, β and γ are parameters satisfying the constraint $\alpha+\beta+\gamma = -1$ and V the potential energy. Clearly there are different Hamiltonians depending on the choices of the parameters. Here we shall work with the BenDaniel-Duke form which corresponds to the choice $\alpha = \gamma = 0$, and $\beta = -1$ [12] such as

$$\hat{H} = \frac{1}{2}\hat{p}\frac{1}{m(\hat{x})}\hat{p} + V(\hat{x}). \tag{2}$$

The corresponding Schrödinger equation for a particle in square well potential i.e. $V(x) = 0$ for $0 < x < L$ is given by

$$E\psi(x,t) = -\frac{\hbar^2}{2m_0}\left(\sqrt{\frac{m_0}{m(x)}}\partial_x\right)^2\psi(x,t) \tag{3}$$

where m_0 is a constant mass. For the present case, we choose the PDM $m(x)$ profile in [6] as

$$m(x) = \frac{m_0}{(1-\tau x+\tau^2x^2)^2}, \tag{4}$$

where τ is deformed parametr $0 < \tau < 1$. The solution of the equation (3) is given by [6]

$$\psi_k(x) = A\exp\left(i\frac{2k}{\tau\sqrt{3}}\left[\arctan\left(\frac{2\tau x-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]\right), \tag{5}$$

where $k = \frac{\sqrt{2mE}}{\hbar}$ and

$$A = \sqrt{\frac{\tau\sqrt{3}}{2}}\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]^{-1/2}. \tag{6}$$

Based on the reference [6], the scalar product of the formal eigenstates is given by

$$\langle\psi_{k'}|\psi_k\rangle = \frac{\tau\sqrt{3}}{2(k-k')\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)\right]}\sin\left(\frac{2(k-k')\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)\right]}{\tau\sqrt{3}}\right). \tag{7}$$

We suppose that the wave function satisfies the Dirichlet condition i.e it vanishes at the boundaries $\psi(0) = 0 = \psi(L)$. Thus, using especially the boundary condition $\psi(0) = 0$, the above wavefunctions (5) becomes

$$\psi_k(x) = A\sin\left(\frac{2k}{\tau\sqrt{3}}\left[\arctan\left(\frac{2\tau x-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]\right). \tag{8}$$

The quantization follows from the boundary condition $\psi(L) = 0$ and leads to the equation

$$\frac{2k_n}{\tau\sqrt{3}}\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right] = n\pi \quad \text{with} \quad n\in\mathbb{N}^*, \tag{9}$$

$$k_n = \frac{\pi\tau\sqrt{3}n}{2\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]}. \tag{10}$$

Then, the energy spectrum of the particle is written as

$$E_n = \frac{3\pi^2\tau^2\hbar^2 n^2}{8m_0\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]^2}. \tag{11}$$

At the limit $\tau \to 0$, we have

$$\lim_{\tau\to 0} E_n = \varepsilon_n = \frac{\pi^2\hbar^2 n^2}{2m_0 L^2}, \tag{12}$$

where ε_n is the spectrum of a free particle in an infinite square well potential of the basic quantum mechanics with the fundamental energy $\varepsilon_1 = \frac{\hbar^2\pi^2}{2m_0 L^2}$. Thus, the energy levels can be rewritten as

$$E_n = \frac{3}{4}\left[\frac{\tau L}{\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}}\right]^2 \varepsilon_n < \varepsilon_n. \tag{13}$$

The generalized wave function and the probability density corresponding to the energies (11) are given by

$$\psi_n(x) = A\sin\left(\frac{n\pi}{\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]}\left[\arctan\left(\frac{2\tau x-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]\right), \tag{14}$$

$$\eta_n(x) = A^2\sin^2\left(\frac{n\pi}{\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]}\left[\arctan\left(\frac{2\tau x-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]\right). \tag{15}$$

At the limit $\tau \to 0$, we have

$$\lim_{\tau\to 0}\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right), \tag{16}$$

$$\lim_{\tau\to 0}\eta_n = \frac{2}{L}\sin^2\left(\frac{n\pi}{L}x\right). \tag{17}$$

3 GK coherent states for a position-dependent mass

The GK-coherent states [7] for a Hermitian Hamiltonian $\hat{H}$ with discrete, bounded below and nondegenerate eigenspectrum are defined as a two parameter set

$$|J,\gamma\rangle = \frac{1}{\mathcal{N}(J)}\sum_{n=0}^{\infty}\frac{J^{\frac{n}{2}}e^{-i\gamma e_n}}{\sqrt{\rho_n}}|\psi_n\rangle, \tag{18}$$

where $J \in \mathbb{R}^+$, $\gamma \in \mathbb{R}$. The states $|\psi_n\rangle$ are the orthogonal eigenstates of $\hat{H}$, that is $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle = e_n|\psi_n\rangle = \omega n^2|\psi_n\rangle$ with $\omega > 0$ and $0 = e_0 < e_1 < e_2 \ldots$. The normalization constant $\mathcal{N}(J)$ is chosen so that

$$\langle\gamma, J|J,\gamma\rangle = \mathcal{N}^{-2}(J)\sum_{n=0}^{\infty}\frac{J^n}{\rho_n} = 1. \tag{19}$$

Thus

$$\mathcal{N}^2(J) = \sum_{n=0}^{\infty}\frac{J^n}{\rho_n}. \tag{20}$$

The allowed values of J, $0 < J < R$ are determined by the radius of convergence $R = \lim_{n\to\infty}(\rho)^{1/n}$ in the series defining $\mathcal{N}^2(J)$. The moments of a probability distribution ρ_n is given by

$$\rho_n = \int_0^R x^n \rho(x)dx = \prod_{k=1}^{n} e_k, \quad \rho_0 = 1. \tag{21}$$

We recover from the coherent states (18), the usual canonical coherent states by setting $\alpha = \sqrt{J}e^{-i\gamma}$ and $\mathcal{N}^{-1}(J) = e^J$ [7] given by

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |\psi_n\rangle. \tag{22}$$

The GK-coherent states (18) have to satisfy the following properties [7–9]:

1. Normalizability: $\langle\gamma, J|J, \gamma\rangle = 1$,
2. Continuity: the mapping $|J', \gamma'\rangle \to |J, \gamma\rangle$ is continuous in some appropriate topology.
3. Resolution of unity: $\int |J, \gamma\rangle\langle\gamma, J|d\mu(J, \gamma) = \mathbb{I}$.
4. Temporal stability: $e^{-iHt}|J, \gamma, \rangle = |J, \gamma + \omega t\rangle$, with $\omega = \text{const}$.

3.1 Construction of GK coherent states

For the system under consideration, the energy eigenvalues defined in Eq.(11), can be obtained as:

$$E_n = e_n = \omega n^2 \tag{23}$$

where

$$\omega = \frac{3\pi^2\tau^2\hbar^2}{8m\left[\arctan\left(\frac{2\tau L-1}{\sqrt{3}}\right) + \frac{\pi}{6}\right]^2} \tag{24}$$

The parameter $\rho(n)$ is defined as

$$\rho(n) = \omega^n (n!)^2, \quad \rho_0 = 1. \tag{25}$$

The GK coherent states given in equation (18) may be expressed as follows

$$|J, \gamma\rangle = \frac{1}{\mathcal{N}(J)} \sum_{n=0}^{\infty} \frac{J^{\frac{n}{2}} e^{-i\gamma e_n}}{\sqrt{\rho_n}} |\psi_n\rangle \tag{26}$$

By multiplying the above equation by the vector $\langle x|$, we express the coherent states (26), in term of the discrete wave function (14)

$$\psi_n(x, J, \gamma) = \frac{1}{\mathcal{N}(J)} \sum_{n=0}^{\infty} \frac{J^{\frac{n}{2}} e^{-i\gamma e_n}}{\sqrt{\rho_n}} \psi_n(x) \tag{27}$$

and the corresponding probability density is given by

$$\eta_n(x, J, \gamma) = \frac{1}{\mathcal{N}^2(J)} \sum_{n=0}^{\infty} \frac{J^n}{\rho_n} \eta_n(x). \tag{28}$$

3.2 Mathematical properties

In this subsection, we discuss the mathematical properties of these states (26) by analysing the non-orthogonality, the conditions of continuity in the label, normalizability, the resolution of identity by finding the weight function $\mathcal{W}(J)$ and the temporal stability.

3.2.1 The non-orthogonality

In order to characterize these states, we can see that the scalar product of two coherent states does not vanish

$$\langle J', \gamma' | J, \gamma \rangle = \frac{1}{\mathcal{N}(J')\mathcal{N}(J)} \sum_{n=0}^{\infty} \frac{e^{-i(\gamma-\gamma')e_n}}{(n!)^2} \frac{(JJ')^{\frac{n}{2}}}{\omega^n}. \tag{29}$$

For $J' = J$ and $\gamma' = \gamma$, the above relation provides us the normalization condition $\langle J, \gamma | J, \gamma \rangle = 1$ of these coherent states and the factors $\mathcal{N}(J)$ come out to be

$$\mathcal{N}^2(J) \quad = \quad \sum_{n=0}^{\infty} \frac{1}{n!} \frac{(J/\omega)^n}{n!} = {}_0F_1\left(1; \frac{J}{\omega}\right), \tag{30}$$

where ${}_0F_1$ is the hypergeometric function and the radius of convergence turns out to be

$$R = \lim_{n\to\infty} [\omega^n (n!)^2]^{1/n} = \infty. \tag{31}$$

3.2.2 The Label continuity

The label continuity condition of the $|J, \gamma\rangle$ can then be stated as,

$$|||J'\gamma'\rangle - |J, \gamma\rangle||^2 = 2[1 - \mathcal{R}e(\langle J', \gamma | J', \gamma' \rangle)] \to 0, \quad \text{when} \quad (J', \gamma') \to (J, \gamma). \tag{32}$$

3.2.3 Resolution of unity

The overcompleteness relation reads as follows

$$\int d\mu(J, \gamma) |J, \gamma\rangle\langle J, \gamma| = \mathbb{I}, \tag{33}$$

where the meseare $d\mu(J, \gamma) = \mathcal{W}(J) \frac{dJd\gamma}{2\pi}$ [10,13]. By substituting equations (26) into equation (33), we obtain

$$\int_0^{\infty} \overline{\mathcal{W}}(J) J^n dJ = [\Gamma(n+1)]^2 \omega^n \tag{34}$$

where $\overline{\mathcal{W}}(J) = \mathcal{W}(J)/\mathcal{N}^2(J)$. Perfoming $n = s-1$, the integral from the above equation is called the Mellin transform

$$\int_0^{\infty} \overline{\mathcal{W}}(J) J^{s-1} dJ = [\Gamma(s)]^2 \omega^{s-1}. \tag{35}$$

Using the definition of Meijers G-function, it follows that

$$\begin{aligned} &\int_0^{\infty} dx x^{s-1} G^{m,n}_{p,q}\left(\alpha x \Big|^{a_1,\cdots,\ \ a_n,\ \ a_{n+1},\ \ \cdots, a_p}_{b_1,\cdots,\ \ b_m,\ \ b_{m+1},\ \ \cdots, b_q}\right) \\ &= \frac{1}{\alpha^s} \frac{\prod_{j=1}^{m} \Gamma(b_j + s) \prod_{j=1}^{n} \Gamma(1 - a_j - s)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j - s) \prod_{j=n+1}^{p} \Gamma(a_j + s)}. \end{aligned} \tag{36}$$

Comparing equations (35) and (36), we obtain that

$$\overline{\mathcal{W}}(J) = \frac{1}{\omega} G^{2,0}_{0,0}\left(\frac{J}{\omega}\middle|\begin{matrix}\cdots\\ 0,\ 0\end{matrix}\right). \tag{37}$$

Since $\overline{\mathcal{W}}(J) = \mathcal{W}(J)/\mathcal{N}^2(J)$, we finally get

$$\mathcal{W}(J) = \frac{{}_0F_1\left(1; \frac{J}{\omega}\right)}{\omega} G^{2,0}_{0,0}\left(\frac{J}{\omega}\middle|\begin{matrix}\cdots\\ 0,\ 0\end{matrix}\right) \tag{38}$$

3.2.4 The temporal stability

The time evolution of coherent states $|J,\gamma\rangle$ can be obtained by unitary transformation $|J,\gamma,t\rangle = \hat{U}(t)|J,\gamma\rangle$ where the time evolution operator is given as $\hat{U}(t) = e^{-i\hat{H}t}$. In the present case, the time evolution of the GK coherent states (26) is given by

$$|J,\gamma,t\rangle = \frac{1}{\mathcal{N}(J)} \sum_{n=0}^{\infty} \frac{J^{\frac{n}{2}} e^{-ie_n(\gamma+\omega t)}}{\sqrt{\rho_n}} |\psi_n\rangle = |J,\gamma+\omega t\rangle. \tag{39}$$

By multiplying the above equation by the vector $\langle x|$, we have

$$\psi_n(x,J,\gamma,t) = \frac{1}{\mathcal{N}(J)} \sum_{n=0}^{\infty} \frac{J^{\frac{n}{2}} e^{-ie_n(\gamma+\omega t)}}{\sqrt{\rho_n}} \psi_n(x). \tag{40}$$

The probability density in space and time as well is

$$\begin{aligned} \eta_{nm}(x,\gamma,J,t) &= \psi_n(x,J,\gamma,t)\psi^*_m(x,J,\gamma,t) \\ &= \frac{1}{\mathcal{N}^2(J)} \sum_{n,m=0}^{\infty} \frac{J^{\frac{n+m}{2}} e^{-i(e_n-e_m)(\gamma+\omega)t}}{\sqrt{\rho_n\rho_m}} \psi_n(x)\psi^*_m(x). \end{aligned} \tag{41}$$

3.3 The statistical properties

In the present subsection, we investigate some of the quantum statistical properties of these states (26), such as the probability distribution, the mean number of photons, the intensity correlation function and the Mandel parameter.

3.3.1 The probability distribution, the intensity correlation function and the Mandel parameter

The probability of finding the n^{th} photons in the states $|J,\gamma\rangle$ is given by

$$\begin{aligned} P_n &= |\langle\psi_n|J,\gamma\rangle|^2 \\ &= \frac{(J/\omega)^n}{\mathcal{N}^2(J)(n!)^2}. \end{aligned} \tag{42}$$

The intensity correlation function or equivalently the Mandel Q-parameter yields the information about photon statistics of the quantum states. The intensity correlation function of the states (26) is defined by

$$g^{(2)}(0) = \frac{\langle\hat{N}^2\rangle - \langle\hat{N}\rangle}{[\langle\hat{N}\rangle]^2} \tag{43}$$

where N is the number operator which is defined as the operator which diagonalizes the basis for the number states

$$\hat{N}|\psi_n\rangle = n|\psi_n\rangle \tag{44}$$

The Mandel Q-parameter is related to the intensity correlation function by

$$Q = \langle\hat{N}\rangle[g^{(2)}(0) - 1]. \tag{45}$$

The intensity correlation function (or the Mandel Q-parameter) determines whether the GKCSs have a photon number distribution. This latter is sub-Poissonian if $g^{(2)}(0) < 1$ (or $-1 \leqslant Q < 0$), Poissonian if $g^{(2)}(0) = 1$ or $(Q = 0)$, and super-Poissonian if $g^{(2)}(0) > 1$ (or $Q > 0$).

We check that, for GKCSs (26), the expectation values of $\hat{N}$ and $\hat{N}^2$ can be computed as

$$\langle\hat{N}\rangle = \langle J,\gamma|\hat{N}|J,\gamma\rangle = \frac{1}{\mathcal{N}^2(J)}\sum_{n=0}^{\infty}\frac{(J/\omega)^n}{(n!)^2}n = \frac{J}{\omega}.\frac{{}_0F_1\left(2;\frac{J}{\omega}\right)}{{}_0F_1\left(1;\frac{J}{\omega}\right)}, \tag{46}$$

$$\langle\hat{N}^2\rangle = \langle J,\gamma|\hat{N}^2|J,\gamma\rangle = \frac{1}{\mathcal{N}^2(J)}\sum_{n=0}^{\infty}\frac{(J/\omega)^n}{(n!)^2}n^2 = \frac{J^2}{2\omega^2}.\frac{{}_0F_1\left(3;\frac{J}{\omega}\right)}{{}_0F_1\left(1;\frac{J}{\omega}\right)} + \langle\hat{N}\rangle. \tag{47}$$

Taking into account the results (46, 47) of the expectation values of the number operator and its square, one gets

$$g^{(2)}(0) = 0.5\frac{{}_0F_1\left(3;\frac{J}{\omega}\right){}_0F_1\left(1;\frac{J}{\omega}\right)}{[{}_0F_2\left(2;\frac{J}{\omega}\right)]^2}. \tag{48}$$

It is straightforward to remark that $g^{(2)}(0) < 1$ which indicates that the GKCS have sub-Poissonian statistics for all values of J and ω. The Mandel parameter Q is given by

$$Q = \frac{J}{2\omega}\left[\frac{{}_0F_1\left(3;\frac{J}{\omega}\right)}{{}_0F_1\left(2;\frac{J}{\omega}\right)} - 2\frac{{}_0F_1\left(2;\frac{J}{\omega}\right)}{{}_0F_1\left(1;\frac{J}{\omega}\right)}\right] < 0, \tag{49}$$

which confirm that the sub-Poissonian nature of GKCS.

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Dynamics of charged dust near magnetic planets

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Abstract:

In astrophysics, study the dynamics of charged particles in a planetary magnetosphere is very interesting, it describes the region that surrounds a planet and its magnetic field, where these charged components can be trapped and controlled by that planet's magnetic field. In this work, we calculate the trajectories of particles in planetary magnetic field with analytical solutions using a specify mathematical models, then compare the results for magnetic planets to the earth known model using internal sources and spherical harmonics.

Is Generalized Woods-Saxon Potential Supercritical?

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Abstract

Woods-Saxon potential is a famous short-range potential, especially used in Nuclear Physics to describe the dynamics of atoms. Generalized Woods-Saxon potential assumes an additional interaction near the surface of the nucleus which is deduced from the derivative of the Woods-Saxon potential. In this presentation, we discuss the solution of the Dirac equation with the Generalized Woods-Saxon potential in one dimension. Regarding this solution, we investigate the supercriticality of the potential energy.

FAULT-TOLERANCE APPROACHES IN SDN

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Abstract

Software-Defined Networking (SDN) is a newly emerging network paradigm that separates the control plane and the data plane which leads to centralized network control. SDN provides a global network view that offers network flexibility and facilitates failover recovery. To ensure network reliability and availability, Fault-Tolerance is required to minimize the consequences of service failure. This presentation discusses different existing SDN Fault-Tolerance approaches. Our study focuses on the approaches in both, Control and Data planes. We are incorporating a systematic classification of these approaches, their limitations, and open challenges, based on which we will propose some solutions. Moreover, future research issues are mentioned as well.

Keywords: Software-Defined Networking (SDN), Fault-Tolerance, failure recovery.

THERMODYNAMICS OF DUNKL-BOSON SYSTEMS

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Abstract

Within the framework of the theory of Dunkl-deformed bosons, Bose--Einstein condensation of a Dunkl-deformed boson system, is investigated. By using a Fock space defined by making a correspondence with the so-called Dunkl differential--difference operators, we obtain the expressions of the partition function, the effective critical temperature and the ground state population in terms of polylogarithm function.

THE DUNKL-DUFFIN-KEMMER-PETIAU OSCILLATOR IN THE NEW TYPE OF EXTENDED UNCERTAINTY PRINCIPLE

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Abstract

In this work, we present the exact solution of DKP equation oscillator subject to the uniform electric field in the context of the new type of the extended uncertainty principle using the displacement operator method. The energy eigenvalues and eigenfunctions are determined. The wave functions are expressed in terms of the associated Laguerre polynomials.

EFFECTS OF GUP ON HUBBLE AND COSMOLOGICAL CONSTANTS

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We study the effects of some Generalized Uncertainty Principles (GUP) on the evolution of the Friedman-Robertson-LeMaitre-Walker (FRLW) universe. We show that the effects on the Hubble constant are polynomial in both matter and radiation dominated eras but are exponential in vacuum dominated one. We find also that the effects are equivalent to those of the cosmological constant and thus they constitute an additional contribution in the acceleration of the expansion of the universe.

Keywords: GUP, FRLW universe, Hubble constant, Cosmological constant

1 Introduction

Modern physics is built on two fundamental theories that are Quantum Theory (QT) and General Relativity (GR). They each govern one end of our world and we are living in the middle looking for a unified vision that reconciles these two patents of our perception of the world. Both theories suffer from the presence of divergences due to the singularities within them. These divergences come essentially from the punctual character of the particle, which is the main field of QT, and from the point definition which is the main constituent of space, that forms the field of GR.

Despite this, both theories are so precise in their respective fields of application and because of that, a lot of work is reduced to search for tiny changes to both theories. One of the paths taken in these attempts is to modify the commutation relations in QT which are inherent in the properties of all wave-like theories. This led us to consider the Generalized Uncertainty Principle (GUP) instead of Heisenberg's one (HUP) and the main goal is to remedy to theoretical insufficiencies found in the usual QT like singularities found in Fermi theory of β-decay [1, 2] (and the references therein). The GUP approaches can also offer the possibility for space discreteness and therefore to the possible quantization of space.

On another side, many high energy theories predict, or result in, the existence of a minimal fundamental length. We mention here Non-Commutative QM which leads to modifications of HUP and generatee a minimal length scale [3]. In String theory, the string is conjectured not to interact at distances smaller than its size, determined by its tension, and this also leads to GUP [4]; In fact, minimum length appears in several scenarios of quantum gravity [5, 6].

In this work, we are interested in the effects of the minimal length, which comes from QT, on the dynamics of the universe, which is the main field of application of GR. We will therefore see the possible consequences of the application of GUP on the Friedman's equations as they are the perfect representation of the evolution of a homogeneous and isotropic universe, which is the main requirement or the minimum priors for all cosmological models [7]. The accepted cosmological standard model is the ΛCDM where Λ is the cosmological constant introduced in Einstein's equations to explain the acceleration of the expansion of the universe [8,9]. We will consider this model without the Λ-term and demonstrate that this contribution appears automatically in Friedman's equations as a consequence of GUP.

Our work will be organized as follows: we will first present the effects of the GUP relations on Friedman's equations in section 2. Then we will study in section 3 the solutions of these modified equations during the different phases of the evolution of the universe. Finally, we end our study in section 4 with the discussion of the different results found as well as the conclusions that we draw from these results.

2 GUP Effects on Friedman Equations:

We choose to use Pedram's version of GUP [11]:

$$[x_i, p_j] = \frac{i\hbar\delta_{ij}}{1-\beta p^2}\frac{1}{1-\alpha x^2} \tag{1}$$

despite the fact that the version of Kempf, Mangano and Mann (KMM) [12] is the most popular:

$$[x_i, p_j] = i\hbar\delta_{ij}\left(1+\beta p^2\right) \tag{2}$$

This choice is guided by the fact that the Petruzziello's relation is a generalisation of the Pedram formulation of GUP [10] which leads to a limitation of the maximum values of the impulsion in addition to the introduction of a minimal value of the lengths; This eliminates the divergences in kinetic energies, which is not the case of the KMM's one [1, 2]. The relation of Pedram is not perturbative and therefore it is valid for all values of the deformation parameter β; we can see that 2 is just the $1st$ order approximation of 1. We also mention that 1 generates the non-commutativity for spatial coordinates and this is another advantage to add to this expression of the GUP [10]. The Pertuzzielo GUP adds also a maximum measurable length scale which can be defined as the size of the observable universe.

To incorporate the quantum deformation 1 in the "classical" Friedman equations, we need a relation linking the classical and quantum worlds (classical means non-quantum). To do so, we use the correspondence between Poisson and Dirac brackets [13–16]:

$$\begin{aligned}
\{x_i, p_j\} &= \delta_{ij} \to [x_i, p_j] = i\hbar\delta_{ij} \\
\to [x_i, p_j]_{GUP} &= \frac{i\hbar\delta_{ij}}{1-\beta p^2}\frac{1}{1-\alpha x^2} \\
\to \{x_i, p_j\}_{GUP} &= \frac{\delta_{ij}}{1-\beta p^2}\frac{1}{1-\alpha x^2}.
\end{aligned} \tag{3}$$

We need the Hamiltonian formalism for the Friedman-Robertson-LeMaitre-Walker (FRLW) universe. For the isotropic and homogeneous FRLW models, the Hamiltonian is [13–15, 17, 18]:

$$\mathrm{H}_E = \frac{2\pi G}{3}N\frac{p^2}{a} + \frac{3}{8\pi G}Nka - N\rho a^3 + \lambda\Pi \tag{4}$$

$a = a(t)$ is the scale factor and it describes the expansion of the universe. It is the only degree of freedom of the system (so we use it instead of x). $N = N(t)$ is the lapse function and it does not play any dynamical role, k is the geometrical parameter of the universe, G is the gravitational constant, ρ is the density, Π is the momenta conjugate of the lapse function (it vanishes because N is constant) and λ is just a multiplicator.

Using the Hamiltonian formalism, we have 3 equations (We choose here $N = 1$):

$$\dot{a} = \{a, \mathrm{H}_E\} = \frac{4\pi G}{3}\frac{p}{a} \tag{5a}$$

$$\dot{p} = \{p, \mathrm{H}_E\} = \frac{2\pi G}{3}\frac{p^2}{a^2} - \frac{3c^2 k}{8\pi G} + 3\rho a^2 + \frac{d\rho}{da}a^3 \tag{5b}$$

$$\dot{\Pi} = \{\Pi, \mathrm{H}_E\} = -\frac{2\pi G}{3}\frac{p^2}{a} - \frac{3c^2}{8\pi G}ka + \rho a^3 = 0 \tag{5c}$$

Combining 5a and 5c gives us the $1st$ Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{6}$$

We use this equation with the conservation equation for the constituents of the universe:

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)\rho\left(1 + \omega c^2\right) = 0 \tag{7}$$

to find the $2nd$ Friedman equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1 + 3\omega c^2\right) \tag{8}$$

ω is defined from the equation of state $P = \omega\rho c^2$ where P is the pressure in the universe when we consider it as a perfect fluid.

Now using the correspondence 3, we get the following modified Hamiltonian equations:

$$\dot{a}_{GUP} = \left(\frac{4\pi G}{3}\frac{p}{a}\right)\frac{1}{1 - \beta p^2}\frac{1}{1 - \alpha a^2} \tag{9a}$$

$$\dot{p}_{GUP} = \{p, \mathrm{H}_E\}\frac{1}{1 - \beta p^2}\frac{1}{1 - \alpha a^2} \tag{9b}$$

$$\dot{\Pi}_{GUP} = \{\Pi, \mathrm{H}_E\}_{GUP} = \{\Pi, \mathrm{H}_E\} = 0 \tag{9c}$$

Here we have modified the Petruzziello formulation by using the scale factor a instead o x in the fraction because a is the dinalical variable in cosmology; so our parameter α is dimensionless whereas the Petruzziello one has a dimension $(length)^{-2}$.

We have to solve 9a for p and then use it in 9c to find the $1st$ modified equation of Friedman; we get:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\left(1 - \alpha a^2\right)\left(1 - \left(\frac{3a^2}{4\pi G}\right)^2 H_0^2\beta\right)\right]^{-2} \tag{10a}$$

$$H^2 \approx H_0^2\left[1 + 2\alpha a + 2\left(\frac{3a^2}{4\pi G}\right)^2 H_0^2\beta\right] \tag{10b}$$

Where H is the Hubble constant and its value without deformations comes from 6:

$$H_0 = H\left(\alpha = 0, \beta = 0\right) = \sqrt{\frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}} \tag{11}$$

eq.10b is writing to the 1st order in α and β.

We obtain the 2nd Friedman equation by using the conservation equation 7 with the 1st one 10a (or 10b) and the 2nd Hamiltonian equation 9b, so we get:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1+3\omega c^2\right) + \frac{8\pi G}{3}\rho\left(1-3\omega c^2-\frac{3}{4\pi G\rho}\frac{kc^2}{a^2}\right) - 2\left(\frac{3a^2}{4\pi G}\right)^2\left(8\pi G\omega\rho c^2+\frac{kc^2}{a^2}\right)H_0^2\beta \quad (12)$$

As it should be, the two deformed equations are reduced to the usual ones when the deformations are canceled.

We can also easily compare them to the Friedman equations with the cosmological constant term in order to incorporate dark energy contribution [8, 19]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}\right) + \frac{\Lambda c^2}{3} \quad (13a)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1+3\omega c^2\right) + \frac{\Lambda c^2}{3} \quad (13b)$$

Looking for common term coming from the deformations contributions in the two deformed equations 10b and 12, we write:

$$\frac{\Lambda c^2}{3} = 2\left(\frac{4\pi G}{3}a^2\rho - 2kc^2\right)\alpha + 9\frac{k^2c^4}{8\pi^2G^2}\beta \quad (14)$$

We see that the cosmological constant is proportional to the deformations parameters α and β and this can be considered as a justification for the extreme smallness of Λ [20]. It should be noted that for a flat universe (where $\kappa = 0$), the minimal length contribution (proportional to β) vanishes and we are left with the maximal length one:

$$\Lambda = 6\left(\frac{4\pi G}{3c^2}a^2\rho\right)\alpha \quad (15)$$

We can also express the deceleration parameter using the two deformed equations 10b and 12:

$$q_\beta = -\left(\frac{\ddot{a}}{a}\right)\left(\frac{a}{\dot{a}}\right)^2 \quad (16)$$

$$= q_0\left[1 - 2\frac{a^2}{q_0}\alpha - 2\left(\frac{3a^2}{4\pi G}\right)^2 H_0^2\left(\frac{1}{q_0}-1\right)\beta\right] \quad (17)$$

Where q_0 is the usual deceleration parameter without deformations. Because $q_0 \approx 0.5$, we see that the α and β terms decrease the value of the parameter q ($q_\beta < q_0$) and this means that the GUP deformation accelerates the expansion if the universe even if we consider q flat universe as in the ΛCDM model. This is due to the presence of corrective terms in both 10a and 10b without the parameter k.

3 GUP Effects on Friedman Solutions:

In this section, we solve the modified Friedman equation 10a for the different possible domination concerning the densities of the components of the universe and we consider a flat universe. We will follow the chronological order and start with a radiation dominated universe (which is the case of the early stages), then we follow with a matter dominated universe and finally we end with a universe dominated with dark energy as it is considered now.

3.1 Solutions for Radiation Dominated Universe

The conservation relation 7 gives a density proportional to a^{-4} in this case. Writing $\rho_r = \rho_{r,0}a^{-4}$ in 10a and solving for a gives the following solution of the Friedman equation:

$$a(t) = \sqrt{H(t_0}\left(\frac{2t}{t_0}\right)^{1/2}\left(1+\frac{a_0^2}{4}\frac{t}{t_0}\alpha+\frac{9a_0^4}{8^3\pi^2G^2}\beta\right) \tag{18}$$

here a_0 is the scale factor at time t_0 for vanishing deformation parameters.

We see from 18, that the two parameters α and β accelerate the expansion of the universe in a polynomial way in this case ($\propto t^{1/2}$). We use this new expression of the scale parameter 18 to write the Hubble constant:

$$H_\beta(t) = \frac{\dot{a}}{a} = \frac{1}{2t}+\frac{a_0^2}{4}\alpha \tag{19}$$

We see here that additional contribution comes from the maximum length parameter and so, the influence of the minimal length parameter is irrelevant in the value of the Hubble constant at the early stages of the universe. We even made the calculations up to the 2nd order for the deformation parameters and we get a Hubble constant that does not depend on β.

3.2 Solutions for Matter Dominated Universe

The conservation relation 7 gives a density proportional to a^{-3} in this case. We write $\rho_m = \rho_{m,0}a^{-4}$ in 13a and we solve for a to get:

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{2/3}\left[1+\frac{6a_0^2}{21}\left(\frac{t}{t_0}\right)^{4/3}\alpha+\frac{3a_0^6}{5\pi^2G^2}\left(\frac{3}{2}\frac{t}{t_0}\right)^{2/3}\beta\right] \tag{20}$$

The deformations effects in this case are also polynomial ($\propto t^{4/3}$).

For the Hubble constant, we use this new formula of $a(t)$ to find:

$$H_\beta(t) = \frac{2}{3t}+\frac{8a_0^2\alpha}{21}\left(\frac{t}{t_0}\right)^{1/3}+\frac{8a_0^6\beta}{5}\left(\frac{3}{8\pi G}\right)^2\left(\frac{2t_0}{3t}\right)^{1/3} \tag{21}$$

The corrections here are polynomial too but the effects coming from the maximum length increase with time (proportional to $t^{1/3}$) while those of the minimum length decrease (inversely proportional to $t^{1/3}$).

3.3 Solutions for Vacuum Dominated Universe

The conservation relation 7 gives a constant density in this case. So the solution of the deformed Friedman equation is:

$$a(t) = a_0e^{H_0(t-t_0)}\left[1+e^{2H_0(t-t_0)}\alpha+\frac{3\rho_{v,0}H_0^3}{8\pi G}e^{4H_0(t-t_0)}\beta\right] \tag{22}$$

For the Hubble constant, we get:

$$H_\beta(t) = H_0+2H_0e^{2H_0(t-t_0)}\alpha+\frac{3\rho_{v,0}H_0^3}{2\pi G}\beta e^{4H_0(t-t_0)} \tag{23}$$

$H_0 = \sqrt{8\pi G\rho/3}$ is the actual value of the Hubble constant without deformation (we remind that $k=0$).

We see that the corrections coming from the two parameters are exponential which differs from the forms found in the two previous eras.

4 Conclusion:

We have studied in this work the effects of the GUP with the two minimum and maximum lengths on the evolution of the FRLW universe. We demonstrated that GUP effects on both scale factor and the Hubble constant are polynomial and for matter dominated universe while they are exponential in the vacuum epoch. In the radiation era, we found that only the maximal length effects are noticeable. These effects coming from the maximal length parameter are inversely proportional to the time in matter dominated universe that is the era of the Cosmic Microwave Background (CMB) while they are exponential in the actual era where the universe is dominated by vacuum energy. This fact can contribute to the explanation of the difference which exists between the numerical values of the constant of Hubble [21].

When we look at the effects on the deceleration parameter, we see that the two deformations contribute to the acceleration of the expansion of the universe. This is also visible in the deformed Friedmann equations, where we see that the deformations add terms that are equivalent to the term of the cosmological constant in the ΛCDM model of the universe. But it should be noted here that the deformation due to the minimal length only contributes in the case of a non-flat universe. These corrections coming from the deformation terms can contribute to solve the problem of the enormous difference between the experimental value and the theoretical value of the cosmological constant or what is commonly called "the worst theoretical prediction in the history of physics!" [22].

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ANOMALOUS TRANSPORT IN COMPLEX SYSTEMS

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Abstract

Transport dynamics in complex or disordered media namely anomalous transport, as in amorphous semiconductors and insulators, is represented and derived through the mathematical instruments of stochastic mechanics and fractional calculus. Anomalous transport can be described through the drift-diffusion equations that are reformulated in the fractional form to indicate anomalous diffusion and fractional drift processes. Random walk in a random environment and continuous-time random walk (CTRW) are analogical stochastic processes that are used to mimic anomalous transport in complex media. Fractional dynamics has also a critical role in modeling anomalous transport in complex media that exhibits non-Markovian (not memoryless) and non-local properties. There are numerous master equations (MEs) such Klein-Kramers (KKE), Fokker-Planck (FPE), Telegraph Equation (TE), Diffusion-Advection (DAE) equations that each express different stochastic processes occurring in anomalous transport phenomena. The analytical solution of the fractional master equations for the corresponding transport problems can be obtained using the Fourier-Laplace (FL) transform that allows changing the space-time domain with the FL domain. The anomalous transport in complex or disordered media that can be defined in amorphous semiconductors and insulators is governed through analytically derived Debye and semi-empirically derived Cole-Cole (CC), Cole-Davidson (CD) and Havriliak-Negami (HN) type conductivity equations. The common contemporary motivation is to construct the relationship between the semi-empirical conductivity equations especially HN type and the anomalous transport processes governed by the approaches of fractional and stochastic dynamics.

JOULE-THOMSON EXPANSION IN BLACK HOLE THERMODYNAMICS

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Abstract

We review the thermodynamical properties of a black hole in various cases. A black hole has entropy and temperature but its entropy is proportional to the area of the event horizon of a black hole. Nevertheless the source of black hole entropy is puzzling. This problem is not an obstacle to research the properties and processes in which a black hole undergoes. A black hole exhibits crtical behavior as a van der Waals gas so it presents phase transtion of the Schwarxschild black hole. We also revisit the Joule-Thomson expansion of a blak hole. Eventually we mention and criticize some problematic approaches on the topic.

SPECTROSCOPY OF DOUBLY HEAVY Ω_{cc} BARYON IN THE NONRELATIVISTIC QUARK MODEL

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In this study, we computed the ground state and the excited state masses of doubly heavy Ω_{cc} baryons in the nonrelativistic quark model. First, we have analytically solved the radial Schrödinger equation with the hypercentral potential by using the ansatz method. We considered the first order correction and the spin-dependent part to the hypercentral potential. Our results are compared with other theoretical reports, which could be a benefits tool for the interpretation of experimentally unknown doubly heavy baryons spectrum.

Keywords: Heavy Baryons, Hypercentral Potential, Mass Spectrum, Ansatz Method, Nonrelativistic Quark Model.

1. Introduction

Baryon spectroscopy is a key to strong interactions in the region of quark confinement and very beneficial for understanding the baryon as a bound state of quarks and gluons. The bound state heavy baryons can be studied in the QCD motivated potential models treating to the nonrelativistic Quantum mechanics. A Constituent Quark Model is a modelization of a baryon as a system of three quarks or anti-quarks bound by some kind of confining interaction. This study deals with the Hypercentral Constituent Quark Model, an effective way to investigate three body systems is through consideration of Jacobi coordinates. Doubly heavy baryons, with two heavy quarks and one light quark, are expected to exist in QCD and their masses have been predicted in the quark model. Ω_{cc} baryon has a light strange quark with two heavy c quarks [1]. In recent years experiments and theoretical outcomes have been used in studying the heavy baryons. The experimental results have been reported by different experimental facilities like CLEO, Belle, BaBar, LHCb, etc [2, 3] on ground states and many new excited states of heavy flavour baryons. The particle Data Group (PDG) listed20known charm baryons in their PDG [4]. The experimental evidence comes for Ξ_{cc}^{+} (containing two charm quarks) with a mass of ~3520 MeV/c^2 by the SELEX experiment and LHCb has determined the ground state of Ξ_{cc}^{++} baryon [4-7]. For study the doubly heavy baryons, we have used the hypercentral constituent quark model (hCQM) with the hypercentral potential We also added the first order correction and the spin-dependent part to the potential. We have obtained the mass spectra of radial excited states up to 5S and orbital excited states for 1P-5P, 1D-4D and 1F-2F states.

This paper is organized as follows: A description of the Hypercentral Constituent Quark Model, the interaction potentials between three quarks and the exact analytical solution of the radial Schrödinger equation for our proposed potential are given in section 2. In sect. 3, our results are given and compare with other predictions. Finally, we draw conclusions in section 4.

2. Theoretical Model

The Hypercentral Constituent Quark Model has successfully given the mass spectroscopy of the baryons in heavy sector. The relevant degrees of freedom for the motion of heavy quarks are related by the Jacobi coordinates, ρandλ [8] and the respective reduced masses are given by m_ρ and m_λ . In order to describe three-quark dynamics, we define hyper radius $x$and hyper angle ξ in terms of the absolute values ρ and λ of the Jacobi coordinates [9]. In this paper, the confining three-body potential is regarded as

$$V(x) = V^{(0)}(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right) V^{(1)}(x) + V_{SD}(x) \tag{1}$$

where $V^{(0)}(x)$ is given by:

$$V^{(0)} = Ax^{-1} + Bx^2 + Cx^{-3} + Dx^{-4} \tag{2}$$

Where *A*, *B*, *C* and *D* are constants. The first order correction $V^{(1)}(x)$ can be written as [10-12]

$$V^1(x) = -C_F C_A \frac{\alpha_S{}^2}{4x^2} \tag{3}$$

The parameters $C_F = \frac{2}{3}$ and $C_A = 3$ are the casimir charges of the fundamental and adjoint representation. The spin dependent part $V_{SD}(x)$ is given as [13]

$$V_{SD}(x) = V_{SS}(x)(\vec{S}_\rho . \vec{S}_\lambda) + V_{\gamma S}(x)(\vec{\gamma} . \vec{S}) + V_T(x)\left[S^2 - \frac{3(\vec{S}.\vec{x})(\vec{S}.\vec{x})}{x^2}\right] \tag{4}$$

The description of all terms can be found in Refs. [14, 15]. The spectrum masses of baryon are obtained by the sum of the model quark masses plus kinetic energy, potential energy and the spin dependent interaction as [16]

$$M_B = \sum m_i + \langle H \rangle \tag{5}$$

The Hamiltonian for the baryonic system is expressed as [17]

$$H = \frac{P_\rho^2}{2m} + \frac{P_\lambda^2}{2m} + V(x) \tag{6}$$

and the hyperradial Schrödinger equation corresponding to the above Hamiltonian can be written as [18]

$$\left(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right)\psi_{\nu\gamma}(x) = -2m[E - V(x)]\psi_{\nu\gamma}(x) \tag{7}$$

By using the transformation $\psi_{\nu\gamma}(x) = x^{-\frac{5}{2}}\varphi_{\nu\gamma}$ reduces Eq. (7) to the form

$$\phi_\gamma{}''(x) + [2mE - 2m\{Ax^{-1} + Bx^2 + [c + \frac{2A}{3m_\rho m_\lambda}(S_\rho.S_\lambda) + \frac{7As^2}{6m_\rho m_\lambda}]x^{-3} + Dx^{-4} + [\frac{3c}{2m_\rho m_\lambda}(\gamma.S) - \frac{21A(s.x)(s.x)}{6m_\rho m_\lambda}]\,x^{-5} + \frac{4Dx^{-6}}{2m_\rho m_\lambda}(\gamma.S) - \frac{B}{2m_\rho m_\lambda}(\gamma.S) + [\frac{-3A}{2m_\rho m_\lambda}(\gamma.S) + (\frac{1}{m_\rho} + \frac{1}{m_\lambda})(-c_f c_A \frac{\alpha_s^2}{4x^2})]\,x^{-2}\} - \frac{(2\gamma+3)(2\gamma+5)}{4x^2}]\phi_\gamma(x) = 0 \tag{8}$$

We suppose $\phi_\gamma(x) = h(x)e^{g(x)}$ and for the functions $h(x)$ and $g(x)$ we make use of the ansatz [19-21]. By doing some calculations, we will have the energy eigenvalues as

$$E_{\upsilon\gamma} = \frac{m\omega^2}{4m_\rho m_\lambda}(\gamma.S) + \frac{\omega}{2}(\delta - 1) \tag{9}$$

3. Baryons Masses

Finally for calculating the best doubly heavy Ω_{cc} baryons masses, the values of m_s, m_c, α_S, ω and β (which are listed in Table 1) are selected using genetic algorithm.

Table 1. The Quark mass (in GeV) and the fitted values of the parameters used in our calculations.

m_s	m_c	C_F	C_A	β	ω
0.530	1.281	$\frac{2}{3}$	3	0.018	0.161 fm^{-1}

Our predicted for the ground, radial and orbital excited states masses of doubly heavy Ω_{cc} baryons are compared with other predictions in Tables (2-4).

Table2. The outcomes ground state masses of $\Omega_{cc}^{+}$are listed with other theoretical predictions (in GeV).

Baryons	Ω_{cc}^{+}	
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$
Our Calc	3.653	3.751
Ref.[18]	3.650	3.810
Ref.[22]	3.719	3.746
Ref.[23]	3.778	3.872
Ref.[24]	3.648	3.770
Ref.[25,26]	3.710	3.760
Ref.[27,28]	3.730	3.780
Ref.[29]	3.832	3.883
Ref.[30]	3.815	3.876
Ref.[31]	3.697	3.769
Ref.[32]	3.747	3.819
Ref.[33]	3.713	3.785
Ref.[34]	3.738	3.822
Ref.[35]	3.740	3.779
Ref.[36]	3.654	3.724
Ref.[37,38]	3.650	3.809
Ref.[39]	3.702	3.783
Ref.[40]	3.667	3.758
Ref.[41]	3.710	3.800
Ref.[42]	3.740	3.820

Table 3. The masses of radial excited states for Ω_{cc}^{+} (in GeV).

Baryons	State	J^P	Our Calc	[18]	[29]	[30]	[31]	[24]	[23]
Ω_{cc}	2S	$\frac{1}{2}^+$	3.989	4.041	4.227	4.180	4.112	4.268	4.075
		$\frac{3}{2}^+$	3.996	4.096	4.263	4.188	4.160	4.334	4.174
	3S	$\frac{1}{2}^+$	4.191	4.338	4.295			4.714	4.321
		$\frac{3}{2}^+$	4.275	4.365	4.265			4.776	
	4S	$\frac{1}{2}^+$	4.483	4.598					
		$\frac{3}{2}^+$	4.567	4.614					

5S	$\frac{1}{2}^+$	4.811	4.836
	$\frac{3}{2}^+$	4.822	4.845

Table 4. The masses of orbital excited of Ω_{cc}^{+} (in GeV).

State	Our Calc	[18]	[29]	[30]	[33]	[23]	[37]
$(1^2 P_{1/2})$	3.975	3.989	4.086	4.046	4.061	4.002	
$(1^2 P_{3/2})$	3.962	3.972	4.086	4.052	4.132	4.102	3.910
$(1^4 P_{1/2})$	3.989	3.998					
$(1^4 P_{3/2})$	3.973	3.981					
$(1^4 P_{5/2})$	3.936	3.958	4.220	4.152			4.058
$(2^2 P_{1/2})$	4.205	4.273	4.199	4.135		4.251	
$(2^2 P_{3/2})$	4.175	4.259	4.201	4.140		4.345	
$(2^4 P_{1/2})$	4.237	4.280					
$(2^4 P_{3/2})$	4.182	4.266					
$(2^4 P_{5/2})$	4.166	4.247					
$(3^2 P_{1/2})$	4.468	4.529					
$(3^2 P_{3/2})$	4.451	4.517					
$(3^4 P_{1/2})$	4.489	4.536					
$(3^4 P_{3/2})$	4.471	4.523					
$(3^4 P_{5/2})$	4.434	4.506					
$(4^2 P_{1/2})$	4.756	4.767					
$(4^2 P_{3/2})$	4.739	4.755					
$(4^4 P_{1/2})$	4.768	4.772					
$(4^4 P_{3/2})$	4.754	4.761					
$(4^4 P_{5/2})$	4.732	4.745					
$(5^2 P_{1/2})$	5.198	4.989					
$(5^2 P_{3/2})$	5.182	4.978					
$(5^4 P_{1/2})$	5.116	4.994					
$(5^4 P_{3/2})$	5.109	4.984					
$(5^4 P_{5/2})$	5.098	4.969					
$(1^4 D_{1/2})$	4.196	4.186					
$(1^2 D_{3/2})$	4.137	4.162					
$(1^4 D_{3/2})$	4.145	4.170					
$(1^2 D_{5/2})$	4.087	4.141	4.264	4.202			4.153
$(1^4 D_{5/2})$	4.480	4.149					
$(1^4 D_{7/2})$	4.469	4.122					4.294
$(2^4 D_{1/2})$	4.478	4.446					
$(2^2 D_{3/2})$	4.451	4.425					
$(2^4 D_{3/2})$	4.459	4.432					
$(2^2 D_{5/2})$	4.451	4.407					
$(2^4 D_{5/2})$	4.459	4.414	4.299	4.232			
$(2^4 D_{7/2})$	4.418	4.391					

Table 4. Continued.

State	Our Calc	[18]	[29]	[30]	[33]	[23]	[37]
$(3^4 D_{1/2})$	4.723	4.642					
$(3^2 D_{3/2})$	4.704	4.625					
$(3^4 D_{3/2})$	4.713	4.631					
$(3^2 D_{5/2})$	4.701	4.611					
$(3^4 D_{5/2})$	4.708	4.616	4.410				
$(3^4 D_{7/2})$	4.693	4.598					
$(4^4 D_{1/2})$	5.231	4.911					
$(4^2 D_{3/2})$	5.180	4.894					
$(4^4 D_{3/2})$	5.199	4.900					
$(4^2 D_{5/2})$	5.170	4.879					
$(4^4 D_{5/2})$	5.186	4.885					
$(4^4 D_{7/2})$	5.193	4.866					
$(1^4 F_{3/2})$	4.468	4.348					
$(1^2 F_{5/2})$	4.443	4.321					
$(1^4 F_{5/2})$	4.449	4.328					
$(1^4 F_{7/2})$	4.420	4.303					
$(1^2 F_{7/2})$	4.410	4.296					4.383
$(1^4 F_{9/2})$	4.398	4.274					4.516
$(2^4 F_{3/2})$	4.802	4.593					
$(2^2 F_{5/2})$	4.778	4.569					
$(2^4 F_{5/2})$	4.788	4.575					
$(2^4 F_{7/2})$	4.760	4.553					
$(2^2 F_{7/2})$	4.745	4.547					
$(2^4 F_{9/2})$	4.721	4.527					

3. Conclusion

In this work, masses of the ground and excited states of doubly heavy Ω_{cc}^{+} baryons has been calculated using the hypercentral constituent quark model. For this goal we have analytically solved the radial Schrödinger equation under the hypercentral potential by using the ansatz method. The mass spectra prediction using the simple basic constituent quark model is quite similar whit predictions using other theoretical approaches.

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DUNKL-SCHRODINGER EQUATION FOR POSITION DEPENDENT MASS

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Abstract

An algebraic method in which the Schrodinger equation with position-dependent mass can be solved exactly is presented. Dunkl derivatives have been used instead of the usual derivative to solve the problem. Our systematic approach reproduces a number of earlier results and also leads to some novelties. We show that the solutions of the Dunkl-Schrödinger equation with position-dependent mass are not free from the choice of parameters for position-dependent mass. The spectrum and wavefunctions of the system are reported and the allowed values of the Dunkl (μ) and dependent mass (α) parameters are obtained through the sl(2) algebraization.

Keywords: Dunkl derivative; Position Dependent Mass; Quasi-Exactly Solvable (QES); Schrodinger equation; sl(2) Lie algebra; representation theory.

KLEIN GORDON OSCILLATOR UNDER A UNIFORM MAGNETIC FIELD IN ANTI DESITTER SPACE

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Abstract

We analyze the two-dimensional deformed bosonic oscillator equation for charged particles (spin 0 particles) in the presence of a uniform magnetic field. We consider the presence of a minimal uncertainty in momentum caused by the anti-de Sitter model, and we solve the system using the Nikiforov-Uvarov (NU) method. The exact energy eigenvalues and corresponding wave functions for Klein-Gordon are obtained analytically, and we find that the deformed spectrum remains discrete even for large values of the principal quantum number.

1 Introduction

The quantum field theory in curved space via generalizations of the Heisenberg algebra, such as the extended uncertainty principle, is one of the various attempts to include gravity in the quantum world that has attracted a lot of interest (EUP). This extended principle allows gravity to be incorporated into quantum mechanics by accounting for the quantum fluctuations of the gravitational field. The existence of a minimum length scale of the Planck order is one of the effects of this unification [1]. The work of Mignemi[1], who demonstrated that the Heisenberg relations are modified in the (anti-)de Sitter space by adding corrections that are proportional to the cosmological constant, is cited here to demonstrate how we can link this minimum length to a modification of the standard Heisenberg algebra by adding small corrections to the canonical commutation relations and thereby shift their standard algebra.

Two-dimensional (2D) systems that describe the dynamics of a charged particle contained by a powerful and consistent external magnetic field are becoming more and more popular. This fascination is a result of their numerous uses in semiconductor structures [2], chemical physics [3], and molecular vibrational and rotational spectroscopy [4]. As a result, a lot of research has been done on this type of issue in both regular and deformed quantum mechanics. Here, we list some examples of 2D systems that are affected by external fields, including the Schrödinger oscillator [5], the KG particle with a pseudo-harmonic oscillator interaction [6], the Dirac equation [7], the Duffin-Kemmer-Petiau (DKP) equation in the cosmic string background [8], the Schrödinger equation with minimal length in noncommutative phase space [9], the KG oscillator with minimal length in noncommutative space [10].

We are particularly interested in phenomenological models of quantum gravity in this work. In the position space representation for deformed quantum mechanics with EUP, we analyze the KG equation analytically in 2D spaces. For this system, we take into account an oscillator-like interaction and an external, uniform magnetic field.

1. Review of Quantum mechanics relations

We may introduce some relations in the deformed quantum mechanics such the deformed Heisenberg algebra leading to the EUP of Ads space given [11-12]

$$[X_i, X_j] = 0 \,;\, [P_i, P_j] = -i\hbar\lambda\epsilon_{ijk} L_k \,;\, [X_i, P_j] = i\hbar(\delta_{ij} - \lambda X_i X_j) \tag{1}$$

λ is a small positive deformation parameter. L_k is the angular momentum variable, expressed as follows :

$$L_k = \epsilon_{ijk} X_i P_i \tag{2}$$

The usual algebra is written:

$$[L_i, P_j] = i\hbar\varepsilon_{ijk} P_k \,;\, [L_i, X_j] = i\hbar\varepsilon_{ijk} X_k \,;\, [L_i, L_j] = i\hbar\varepsilon_{ijk} L_k \tag{3}$$

The Ads model (1) deformed algebra is defined by the existence of a minimum uncertainty in momentum that is nonzero and gives rise to modified Heisenberg uncertainty relationships:

$$\Delta X_i \Delta P_i \geq \frac{\hbar}{2}(1 + \lambda(\Delta X_i)^2) \tag{4}$$

Also generates a minimum uncertainty in momentum. For simplicity, if we assume isotropic uncertainties $X_i = X$, we get

$$(\Delta p_i)_{min} = \hbar\sqrt{\lambda} \tag{5}$$

The non-commutative operators' $X_i$andP_i as functions of the usual $x_i$and $p_i$operators will be used to satisfy the Ads algebra (1),

$$X_i = \frac{x_i}{\sqrt{1-\lambda r^2}}; P_i = -i\hbar\sqrt{1-\lambda r^2}\,\partial x_i \tag{6}$$

2. Klein Gordon Oscillator In a magnetic field

The (2+1) dimensional Klein–Gordon oscillator in a constant magnetic field is defined by the following equation [13]

$$c^2\left(p - \frac{e}{c}A + im\omega r\right)\cdot\left(p - \frac{e}{c}A - im\omega r\right)\Psi(r) = (E^2 - m^2c^4)\Psi(r) \tag{7}$$

We choose the z-axis as the magnetic field direction and use the Coulomb gauge:

$$A = \frac{1}{2}B \times r = \frac{B}{2}(-y, x, 0)$$

B represents the intensity of the magnetic field.

We use the AdS algebra definition (Eqs. (6)) to rewrite this equation in the deformed momentum space:

$$c^2(p^+ \cdot p^-)\,\Psi(r) = (E^2 - m^2c^4)\Psi(r) \tag{8}$$

With the definitions:

$$p^{\pm} = p' \pm im\omega\frac{r}{\sqrt{1-\lambda r^2}},\; p' = \sqrt{1-\lambda r^2}p - \frac{e}{c}B \times \frac{r}{\sqrt{1-\lambda r^2}} \tag{9}$$

Following a straightforward calculation, we get the following equation:

$$\left[(1-\lambda r^2)p^2 + \eta\frac{r^2}{1-\lambda r^2} + i\hbar\lambda(r\cdot p) - \frac{eB}{c}L_z - \varepsilon\right]\Psi(r) = 0 \tag{10}$$

With the parameters

$$\eta = m^2\omega^2 + \frac{e^2B^2}{4c^2} - \lambda\hbar m\omega \text{ and } \varepsilon = \frac{E^2 - m^2c^4}{c^2} + 2m\omega\hbar \tag{11}$$

In order to obtain the exact solution of eq (10), we use the polar coordinates in position space $(r, \varphi)$and write the solutions in a separate form containing the azimuthal quantum number$l$:

$$\Psi(r) = \exp(il\varphi)R(r) \tag{12}$$

So, Eq. (10) transforms to the following expression:

$$\left[\left(\sqrt{1-\lambda r^2}\frac{d}{dr}\right)^2 + \frac{1-\lambda r^2}{r}\frac{d}{dr} - \frac{l^2(1-\lambda r^2)}{r^2} - \frac{\eta r^2}{\hbar^2(1-\lambda r^2)} + \epsilon\right]R(r) = 0 \tag{13}$$

With $\epsilon = \frac{\varepsilon}{\hbar^2} + \frac{eBl}{c\hbar}$

To solve eq (13), we use the transformations $R(\rho) = \rho^{\mu} g(\rho)$with$\rho = \sqrt{(1-\lambda r^2)}$, and then we utilize charge of variables $= 2\rho^2 - 1$, to get the differential equation

$$\left[\frac{d^2}{ds^2} + \frac{\left(\mu - \frac{1}{2}\right) - \left(\mu + \frac{3}{2}\right)s}{1-s^2}\frac{d}{ds} - \frac{(l^2+\epsilon)s^2 + 2l^2 s - (\epsilon - l^2)}{4(1-s^2)^2}\right] g(s) = 0 \qquad (14)$$

This equation (14) has a known class of differential equation, which leads us to use Nikiforov Uvarov method with polynomial [14].

Hence, we found the expressions of the energy eigenvalues as:

$$E_{n,l} = \pm mc^2 \left[1 - \frac{2\omega\hbar}{mc^2} + \frac{2\hbar}{mc^2}\left\{(2n+l+1)\sqrt{\left(\omega - \frac{\lambda\hbar}{2m}\right)^2 + \tilde{\omega}^2} + \frac{\lambda\hbar}{2m}(4n(n+l+1) + 2l + 1) - \tilde{\omega} l\right\}\right]^{\frac{1}{2}} \qquad (15)$$

Where we have used the definition $\tilde{\omega} = \frac{eB}{2mc} = \frac{\omega_c}{2c}$ with $\omega_c$the cyclotron frequency.

We notice that the energy spectrum of our system contains two corrections for deformation; the first is associated with the oscillator term, and the second increases with the deformation parameterλ. It should be noted here that, according to the n^2dependence of the energy levels, which corresponds to confinement at the high-energy area, our result is equivalent to the energy of a spinless relativistic quantum particle in a square well potential; in our case, the well boundaries are placed at$\pm \frac{\pi}{2\sqrt{\lambda}}$.

An interesting characteristic of the spectrum appears when computing the energy levels spacing; indeed, we find the limit:

$$\lim_{n\to\infty} \Delta E_{n,l} = 2\hbar c\sqrt{\lambda} \qquad (16)$$

This expression shows that, without the effects of the deformed algebra, the energy spectrum of the KG oscillator under a strong magnetic field becomes almost continuous for large values of n. In contrast, this continuous feature of the spectrum disappears and it reduces to a bound spectrum in the deformed case. This asymptotic behavior is described by eq(16) where the spacing is proportional to the deformation parameter λ.

In order to get an upper bound for this λ parameter, we use the s-states of the energies from eq (15) and we expand it up to the first order in λ:

$$E_{n,0} = E_{n,0}^{\lambda=0} + \frac{\hbar^2 c^2}{2E_{n,0}^{\lambda=0}}\left[(2n+1)^2 - \frac{(2n+1)\omega}{\sqrt{\omega^2+\tilde{\omega}^2}}\right]\lambda \qquad (17)$$

We use the experimental results of the electron cyclotron motion in a Penning trap. Here, the cyclotron frequency of an electron trapped in a magnetic field of strength B is $\omega_c = \frac{eB}{m_e}$

(without deformation), so we have $m_e \hbar\omega_c = e\hbar B = 10^{-52} kg^2 m^2 s^{-2}$ for a magnetic field of strength $B = 6T$. If we assume that only the deviations of the scale of $\hbar\omega_c$ can be detected at the level $n = 10^{10}$ and that $\Delta E_n < \hbar\omega_c$ (no perturbation of the n-th energy level is observed)[15], we can write the following constraint:

$$\lambda < 3.36 \times 10^{-4} m^{-2} \tag{18}$$

This leads to the following upper bound of the minimal uncertainty in momentum $\Delta P_{min} = \hbar\sqrt{\lambda} < 2 \times 10^{-36} kgms^{-1}$; it is similar to that obtained in Ref [16].

For the non-relativistic limit, by setting $E = mc^2 + E_{nr}$ with the assumption that $mc^2 \gg E_{nr}$, we write the spectrum of the non-relativistic KGO in the deformed AdS space as:

$$E_{nr} = (2n + l + 1)\hbar\sqrt{\left(\omega - \frac{\lambda\hbar}{2m}\right)^2 + \tilde{\omega}^2 + \frac{\lambda\hbar^2}{2m}(4n(n + l + 1) + 2l + 1)} - \hbar\tilde{\omega}l - \hbar\omega \tag{19}$$

In terms of the variables r and φ, we can now write the general form of the wave function Ψ:

$$\Psi(r,\varphi) = C_n 2^{\frac{l}{2}} \exp(il\varphi)(1 - \lambda r^2)^{\frac{\mu}{2}} (\lambda r^2)^{\frac{l}{2}} P_n^{\left(l,\mu - \frac{1}{2}\right)}(1 - 2\lambda r^2) \tag{20}$$

C_n is obtained by using the normalization condition in the space of the radial wave function and the orthogonality relation of the Jacobi polynomials [17]:

$$C_n = \sqrt{\frac{\lambda}{2^l \pi} \frac{n!\left(2n + \mu + l + \frac{1}{2}\right)\Gamma\left(n + \mu + l + \frac{1}{2}\right)}{\Gamma\left(n + \mu + \frac{1}{2}\right)\Gamma(n + l + 1)}} \tag{21}$$

Conclusion

We investigated the exact solutions of the 2D oscillator with an external uniform magnetic field for the KG equation in the context of deformed quantum mechanics with anti-de Sitter commutation relations in this paper. This AdS deformation results in a nonzero minimal uncertainty in momentum measurement. We used the Nikiforov-Uvarov method to obtain analytical expressions for the system's bound state energies and wave functions. We expressed the system's eigenfunctions and corresponding eigenenergies analytically in terms of Jacobi polynomials, with additional corrections depending on the deformation parameter.λ. Our results show that the deformed spectrum remains discrete even for large values of the principal quantum number, implying that the EUP deformation eliminates the spectrum's degeneracy found in the ordinary case (without deformations). We were able to obtain an experimental limit deformation parameter by doing so.

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ASSOCIATION OF THE EARLY AFTERGLOWS WITH THE GRB EMISSIONS

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Abstract

Since the first hazardous discover of the gamma ray-bursts, where chance plays a role for its first observation in 1967, theorists have focused their efforts on the development of theoretical models making it possible to obtain light curves matching with the observations. Especially if these emissions have delay emissions called the afterglows, which are defined as the results of the external shocks of the fire ball with the environment of the Gamma ray bursts. In this work we will talk exactly about the Association of the early Afterglows with the GRB emissions; basing on synchrotron emission as a main radiation mechanism.

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